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Coordination of cooperative advertising models in a one-manufacturer two-retailer supply chain system ☆

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ABSTRACT

This paper considers cooperative advertising issues of a monopolistic manufacturer with competing duopolistic retailers. Four possible game structures (or power configurations), i.e., Stackelberg–Cournot, Stackelberg–Collusion, Nash–Cournot and Nash–Collusion, are discussed. Under each of four game structures, we develop a decision model for the three partners to design the optimal cooperative advertising policies. Through a comparison among the four models, we reveal how cooperative advertising policies and profits of all participants are affected by various competitive behaviors, and then determine whether the partners have any incentives to transit to a different structure. Also presented in the paper are a centralized decision model and a proposed cost-sharing contract, which is able to achieve perfect coordination of the considered channel, where the utility of risk preference is used to determine the fraction of local advertising costs shared by the manufacturer.

1. Introduction

Cooperative (co-op) advertising is an arrangement between one manufacturer and one retailer, where the retailer is responsible for local advertising expense with a portion of it paid by the manufacturer. Generally, the co-op advertising funds are mainly used by two parties for newspaper and magazine ads, brochures, radio and television commercials, even special product displays, etc. so that short-term sales can be improved. After World War II, co-op advertising had been extensively used in developed countries like the United States.

Nowadays, with the increasing competition of markets, more and more manufacturers resort to co-op advertising to strengthen the brand name image of their products and motivate immediate sales at the retail level. The automobile industry, for example, is the most common user of co-op advertising (Green, 2000). The US Co-op Advertising Sourcebook published in 2004 identified more than 4000 co-op advertising programs involving 52 different product classes. In addition, it is estimated by the US National Federation of Independent Business that about $50 billion worth of co-op advertising reimbursement per year is offered to the retailers by manufacturers and wholesalers (Kraft & Kamieniecki, 2007). Hence, in order to maximize individual gain, it is both natural and important to know how to determine each party's optimal cooperative advertising policies in the supply chain. Many researchers discussed this issue from different angles (see Section 2). However, according to our knowledge, few considered the channel with multiple retailers or accounted for the competition between different retailers/buyers in the downstream market. While in many practical supply chains, duopolistic or oligopolistic market is frequently encountered.

In this paper, we consider the co-op advertising issues for a two-echelon supply chain consisting of one monopolistic manufacturer and two duopolistic retailers. We explore two types of game structures (manufacturer-Stackelberg and Vertical-Nash) between two echelons in the channel. As for the duopolistic retailers, we assume that they adopt either Cournot or Collusion. Hence, the co-op advertising problem in our paper will be analyzed respectively in the following four settings: Stackelberg–Cournot, Stackelberg–Collusion, Vertical Nash–Cournot and Vertical Nash–Collusion. Before modeling and analyzing these four settings, we will briefly introduce some real business backgrounds to them.

In a practical supply chain, competition between members of different echelons may be implemented under various game structures: leader–follower structure, Vertical-Nash structure, to name a few. For illustrative purposes, we now introduce a supply chain in China's home appliances industry, where Zhuhai Gree Corporation (Gree), one of the top-niche air conditioning makers, sells its

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products through GOME Appliance Co., Ltd. (GOME), a home appliances retail giant in China. In the starting stages of GOME, Gree had complete control over GOME and hold the power to first announce it wholesale price, brand name investment, local advertising allowance, etc. to force the GOME to follow its decision. Therefore, in that time, a typical leader–follower game structure was observed between the two echelons of this air conditioning supply chain. However, the subsequent couple decades saw GOME growing, at an astonishing speed, into the largest home appliances retailer in China. GOME, along with a few other Chinese home appliances retailers like Suning Appliance Co., Ltd. (Suning), gradually seizes the control of the major sales channels, especially in big cities. These lead to GOME rising in its power and levelling up with Gree to demand advertising allowance and other rights. It is not difficult to recognize an evolving Vertical-Nash game structure between the two companies in this air conditioning supply chain.

Likewise, between the two retailers in the downstream of the supply chain, there also exist various forms of competition, Cournot being one relatively common in the real world. GOME and Suning mentioned above are two home appliances retailers following Cournot. In an industry where size matters, Suning is ready to open more stores in the future as it raises the ante in a battle with GOME. Wal-Mart and Tesco, Carrefour and Auchan, etc. are also cases where Cournot game structure applies. Another behavior pursued by the duopolistic retailers in reality is Collusion, which is a non-competitive agreement between rivals. By collaborating with each other, rival firms look to alter the local advertising effort or price of a product to their advantage. Since overt collusion is usually illegal, most collusion behaviors between two retailers are confidential, including secret agreement, tacit agreement or price alliance, etc. Collusion among retailers, again, is not rare in China. The Beijing-based Guotong Electrical Appliance Company, for example, announced that it has reached an agreement with Asia Financial Service Company. Recently, GOME also revealed its plan to start negotiations on cooperating with its foreign counterparts of which Best Buy Inc. is believed to be a possible choice.

Our main objective of this paper is twofold. One is to reveal how cooperative advertising strategies and the profit of each participant are affected by various competitive behaviors between the two echelons as well as those between the two retailers, and then determine whether the partners have any incentives to transit to a different structure; the other is to introduce a coordinated mechanism that improves both the performance of each party and that of the channel as a whole.

The remainder of the paper is organized as the following. Section 2 provides a review of related literature. Section 3 introduces notations and assumptions. Section 4 shows the decentralized decision models of three members in the two-echelon supply chain. Four non-cooperative game structures are considered and compared. Section 5 presents the centralized decision model among channel members. In Section 6, we present a type of cost-sharing contract to achieve the coordination of the supply chain, and discuss how to design the contract based on the utility of risk preference. The paper concludes with Section 7.

2. Literature review

Berger (1972) was probably the first to analyze co-op advertising issues between a manufacturer and a retailer quantitatively. It was showed in Berger (1972) that mathematical modeling could yield improved managerial decisions and better performance for the whole channel. Berger's model was then extended by researchers in a variety of ways under different co-op advertising settings (see, e.g., Berger, 1973; Berger & Magliozzi, 1992; Dutta, Bergen, John, & Rao, 1995; Fulop, 1988; Somers, Gupta, & Herriott, 1990; Young & Greyser, 1983). Crimmins (1985) pointed out that the co-op advertising is in essence the financial arrangements between two firms, which specify how two firms share the costs caused by the promotion measures. The co-op advertising model presented by Dant and Berger (1996) for a franchising supply chain composed of one franchisor and one franchisee agreed with Berger's model in showing that a joint determination of the franchisor's and the franchisee's advertising expenses could yield more benefit for the channel than sticking to their individual profit-maximization decisions. Roslow, Laskey, and Nicholls (1993) studied co-op advertising in the supply chain and demonstrated that cooperation in advertising investment could increase the profit of the whole supply chain. Khouja and Robbins (2003) investigated the optimal expense of the local advertising and the ordering quantity under the newsboy framework.

However, most of the above literature on co-op advertising considered only the impact of local advertising on the volume of sales, while the influence of other factors, such as the manufacturer's national brand name advertising, was left unaccounted for. Taking the national brand name investment into consideration, Huang, Li, and Mahajan (2002) Li, Huang, Zhu, and Chau (2002) developed independently a cooperative advertising model for a one-manufacturer–one-retailer supply chain. Yet their models, and also those that did not account for the national brand name advertising, assumed a leader–follower relationship in the supply chain between the manufacturer and the retailer. Few considered the market structure in which the retailers' power is equal to or even more than that of the manufacturer. Under both the leader–follower game structure and the simultaneous move game structure, Huang and Li (2001) further discussed the cooperative advertising issue for the supply chain with one manufacturer and one retailer. It is observed in their paper that the manufacturer always prefers the Stackelberg game to the simultaneous move game whereas the preference of the retailer depends on the parameters of the model. Xie and Ai (2006) extended the models developed by Huang and Li (2001) and Li et al. (2002) to the case where the manufacturer's marginal profit is relatively small. By assuming the demand at the retail level was positively related to both local advertising expenses and brand name investment while negatively related to the sales price set by the retailer, Yue, Austin, Wang, and Huang (2006) and Szmerekovsky and Zhang (2009) further extended the work of Huang et al. (2002) and Huang and Li (2001) in developing a price discount model to coordinate the advertising expenses of the two parties. Under the assumption that the consumer demand is dependent on retail price and co-op advertising efforts by channel members, Xie and Neyret (2009) identify the optimal pricing and co-op advertising strategies in four classical types of relationships between a manufacturer and a retailer using the game theory models. By employing a sales response function with respect to advertising expenditures and retail price different from Xie and Neyret (2009), Xie and Wei (2009) further study the Co-op advertising and pricing problems in a one-manufacturer one-retailer channel. A generalization of the earlier works on cooperative advertising by Hempelmann (2006) covered the situation where two parties have asymmetric information on the marginal cost of sales. And Berger, Lee, and Weinberg (2006) presented an optimal cooperative advertising integration strategy for organizations having a direct online channel. They provided integration decisions from a cooperative advertising perspective to determine the profitability of various integration strategies (i.e. "separation" strategy, "partial-integration" strategy and "full-integration" strategy).

Among all of the hitherto literature on cooperative advertising, little discussed a channel where a single manufacturer sells a product through two or more competing retailers. This channel structure, however, represents numerous markets including those made up of specialty stores (e.g., consumer electronics, sporting goods, and automobile parts, to name a few), department stores and supermarkets. Our paper will focus on a channel of this kind, which to be specific, consists of a monopolistic manufacturer and competing duopolistic retailers. The main contributions of the paper are the following. First, this paper extends the current literature related to cooperative advertising to account for a supply chain with multiple retailers. Unlike those in the existing literature, the problem in our paper is discussed and compared under four different settings. Second, this paper takes into account the impact of one retailer's local advertising effort on his rival's sales volume, i.e. the demand one retailer faces is not only related to his local advertising expenditures and the manufacturer's brand name investment, but is also affected by the other retailer's local advertising expenditures. Third, we reveal the influence of different competitive behaviors both between the two echelons and between the two retailers in the downstream market on cooperative advertising policies of all parties.

3. Notations and assumptions

This paper will consider cooperative advertising issues of a two-echelon supply chain in which a monopolistic manufacturer sells its product through duopolistic retailers. The manufacturer invests in the product's national brand name advertising in order to take potential customers from the awareness of the product to the purchase consideration. On the other hand, the manufacturer would like the two retailers to invest in local advertising in the hope of driving potential customers further to the stage of desire and action. Before establishing the models, we give notations and assumptions used in this paper as follows.

3.1. Notations

ρ_m	the manufacturer's dollar marginal profit per unit
ρ_i	the retailer i's dollar marginal profit per unit, $i = 1, 2$
S_i	the demand faced by retailer i ($i = 1, 2$)
m	the national brand name investment of the manufacturer
r_i	the local advertising expenditures invested by retailer i ($i = 1, 2$)
t	the fraction of local advertising expenditures shared by the manufacturer with each retailer[a]
Π_{ri}	profit for retailer i
Π_m	profit for the manufacturer
Π_S	profit for the supply chain system

[a] The reason why we assume that the manufacturer sets the same proportion of the advertising allowance to two competing retailers is threefold. First, the Robinson-Patman Act (1936) requires a manufacturer to treat all competitive retailers equally (proportionately) with respect to advertising allowances. Second, this assumption matches with the reality. For example, Beijing Hualian and Suguo (two large supermarkets in China) receive the same percentage of allowance for the newspaper and magazine ads from the milk manufacturers. Third, it is the tractability requirement.

3.2. Assumptions

(i) The selling prices of the retailers are exogenously determined.

(ii) Generally speaking, the manufacturer's brand name investment and the retailer-i's local advertising perform different yet complementary functions in the sense that they both have positive effects on the product sales of retailer i. However, due to the competitive relationship between two retailers, retailer j's local advertising effort will have a negative impact on the demand retailer i faces (i.e. sales volume; $i \neq j$). This kind of advertisement is referred to as "predatory advertising" in the theory of industrial organization. Therefore, we assume the expected demand S_i of retailer i to be determined by[1]

$$S_i(r_i, r_j, m) = \alpha_i - \beta r_i^{-u} r_j^{v} (1+m)^{-\delta} \quad (i,j = 1,2) \tag{1}$$

where α_i, β, δ, u and v are positive constants. α_i is the maximal potential demand faced by retailer i and $u(v)$ denotes the measure of sensitivity of retailer-i's sales with respect to changes of retailer i's (retailer j's) local advertising expenditures. δ denotes the influence degree of manufacturer's brand name investment on each retailer's demand. It is assumed that $u + v > 1$ and $u > v$. The former is required to assure the existence of the equilibrium solution. The latter means that the demand each retailer faces is more sensitive to his own local advertising efforts than to his rival's. Otherwise, no one would be interested in spending money on the local advertising.

(iii) This paper considers four possible non-cooperative games (or power configurations), which are respectively described as follows.

3.3. Stackelberg–Cournot

In this setting, the manufacturer plays Stackelberg game with the duopolistic retailers, and the manufacturer is the leader and the two retailers the followers; the duopolistic retailers obey Cournot behavior (i.e. play simultaneous move game) in the downstream market of the supply chain.

3.4. Stackelberg–Collusion

In this setting, the manufacturer and the duopolistic retailers play manufacturer-Stackelberg game, whereas the duopolistic retailers pursue Collusion behavior in the downstream market of the supply chain.

3.5. Nash–Cournot

In this setting, the manufacturer and the duopolistic retailers play Vertical-Nash game (i.e. simultaneous move game); the duopolistic retailers obey Cournot behavior in the downstream market of the supply chain.

3.6. Nash–Collusion

In this setting, the manufacturer and the duopolistic retailers play Vertical-Nash game; the duopolistic retailers pursue Collusion behavior in the down stream market of the supply chain.

From notations and assumptions above, we can easily calculate, respectively, the profit functions for the two retailers, the manufacturer and the supply chain system as follows.

$$\Pi_{r1} = \rho_1\left[\alpha_1 - \beta r_1^{-u} r_2^{v}(1+m)^{-\delta}\right] - (1-t)r_1 \tag{2}$$

$$\Pi_{r2} = \rho_2\left[\alpha_2 - \beta r_2^{-u} r_1^{v}(1+m)^{-\delta}\right] - (1-t)r_2 \tag{3}$$

$$\Pi_M = \rho_m\left[(\alpha_1+\alpha_2) - \beta\left(r_1^{-u} r_2^{v} + r_2^{-u} r_1^{v}\right)(1+m)^{-\delta}\right] - t(r_1+r_2) - m \tag{4}$$

$$\Pi_S = \Pi_{r1} + \Pi_{r2} + \Pi_M = (\rho_1+\rho_m)[\alpha_1 - \beta r_1^{-u} r_2^{v}(1+m)^{-\delta}] + (\rho_2+\rho_m)[\alpha_2 - \beta r_2^{-u} r_1^{v}(1+m)^{-\delta}] - r_1 - r_2 - m \tag{5}$$

[1] The reason why the above functions are adopted to depict the retailers' demand is twofold. On one hand, this type of demand form has been successively used in one-manufacturer–one-retailer channel by Huang and Li (2001), Li et al. (2002) and Yue et al. (2006). On the other hand, the theory of industrial organization has pointed out that under the case with two competitive retailers, one party's advertising effort will decrease the other's share of the marketing demand (see Luo, 2006).

4. The decentralized decision models

In the decentralized decision-making system, each entity of the supply chain maximizes its own profit without considering the profit of others. In the following, we will discuss how the manufacturer and the duopolistic retailers determine separately their advertising policies under the four settings mentioned earlier, i.e. Stackelberg–Cournot, Stackelberg–Collusion, Nash–Cournot and Nash–Collusion.

For notational convenience, we use labels "$\hat{}$" and "$^{-}$" over variables to stand for Stackelberg and Vertical-Nash equilibriums between two echelons, respectively; superscripts "*ct*" and "*cn*" to denote *Cournot* and *Collusion* the duopolistic retailers implement, respectively.

4.1. The Stackelberg game between two echelons

In this subsection, we model the relationship between the manufacturer and the duopolistic retailers as a Stackelberg game with the manufacturer being the leader and the retailers the follower. The manufacturer, in order to maximize profit, determines its optimal brand name investment and local advertising allowance, with the latter based on an estimation of the retailers' local advertisement expense. While the two retailers, being the followers, use the information obtained from the manufacturer to set the optimal local advertising expenditure that maximizes their profits.

4.1.1. Stackelberg–Cournot solution

Now assume the two duopolistic retailers adopt the Cournot strategy, where each duopolistic retailer maximizes his own profit on the assumption that the local advertising expenditure set by his rival is invariant with respect to his own decision. Then, in the Stackelberg–Cournot setting, for any the brand name investment m and the shared fraction of local advertising expenditure t declared by the manufacturer, retailer 1 will maximize Π_{r1} given by (2) with respect to r_1, treating r_2 as a parameter, and retailer 2 will maximize Π_{r2} given by (3) with respect to r_2, treating r_1 as a parameter. Since Π_{ri} is a concave function of r_i, retailer i's reaction function can be derived from the first-order conditions of (2) and (3):

$$\partial\Pi_{ri}/\partial r_i = \rho_i\beta u r_i^{-u-1} r_j^{v}(1+m)^{-\delta} - (1-t) = 0 \quad (i,j = 1,2) \tag{6}$$

Solving (6) gives

$$r_1 = \{\rho_1\beta u r_2^{v}/[(1+m)^{\delta}(1-t)]\}^{1/(u+1)} \tag{7}$$

$$r_2 = \{\rho_2\beta u r_1^{v}/[(1+m)^{\delta}(1-t)]\}^{1/(u+1)} \tag{8}$$

From (7) and (8), one can easily derive

$$\partial r_i/\partial t > 0, \partial r_i/\partial m < 0 \text{ and } \partial r_i/\partial r_j > 0 \quad (i,j = 1,2)$$

which can be interpreted into the following property.

Property 1. *(i) A greater fraction of local advertising expenditures shared by the manufacturer would result in an increased amount of local advertising investment by the retailers. (ii) The retailers' local advertising expenditures will decline (increase) if the manufacturer increases (decreases) the level of brand name investment. (iii) A greater local advertising investment of one retailer will lead to the increased spending of the other.*

It can be noted from Property 1 that the manufacturer always wants to adjust m and t, i.e. the brand name investment and the fraction of local advertising shared, in the hope of increasing retailers' local advertising expenditure. Property 1(ii) implies that the retailers will have to spend less money on the local advertising if the manufacturer increases the level of brand name investments. In contrast, if the manufacturer reduces brand name investments, the retailers will enhance local advertising expenses. This philosophy is just consistent with the cockfighting game. And such phenomenon is commonly seen in practice. For example, it was reported by the www.cnscdc.com (the Chinese market investigation website) that the investment of Gree for the television commercials in 2007 was 6.8% higher than that of 2006; however, the expenditure of GOME for printed media (the main outlet for local advertising) in 2007 reduced by 1/5 over the last year. Property 1(iii) reveals the mutual competition between the two retailers in advertising strategies.

Combining (7) and (8), one can derive the optimal local advertising expenditures for each of the retailers as

$$\hat{r}_1^{ct} = (H\beta)^{1/(u-v+1)}(1+m)^{-\delta/(u-v+1)}(1-t)^{-1/(u-v+1)} \tag{9}$$

$$\hat{r}_2^{ct} = \Phi\cdot\hat{r}_1^{ct} \tag{10}$$

where $\Phi = (\rho_2/\rho_1)^{1/(u+v+1)}$ and $H = \rho_1\, u(\Phi)^{v}$.

(9) and (10) present the optimal reaction functions of the two retailers for any given m and t set by the manufacturer. Since the manufacturer is aware of the duopolistic retailers' reaction functions, his profit maximization problem can be formulated by substituting (9) and (10) into (4) as

$$\begin{aligned}\max_{m,t}\Pi_M = {}& \rho_m(\alpha_1+\alpha_2) - [\rho_m\beta(\Phi^{v}+\Phi^{-u}) + H\beta(1+\Phi)t/(1-t)] \\ & \times (H\beta)^{(-u+v)/(u-v+1)}(1+m)^{-\delta/(u-v+1)}(1-t)^{(u-v)/(u-v+1)} - m \\ \text{s.t.}\quad & 0\leqslant t<1,\ m\geqslant 0\end{aligned} \tag{11}$$

By neglecting the constraints and solving $\partial\Pi_M/\partial t = 0$ and $\partial\Pi_M/\partial m = 0$, one can easily obtain a unique solution, denoted by (t_0, m_0), as follows:

$$t_0 = 1 - (1+\Phi)/\{[(\Phi^{v}+\Phi^{-u})\rho_m/H - (1+\Phi)](u-v)\} \tag{12}$$

$$m_0(t_0) = [(H\beta)^{1/(u-v+1)}(1+\Phi)(1-t_0)^{-1/(u-v+1)}\delta \,/(u-v)]^{(u-v+1)/(\delta+u-v+1)} - 1 \tag{13}$$

With taking the constraints into account and defining $F_1(\rho_1, \rho_2) = (1+\Phi)H/(\Phi^{v}+\Phi^{-u})$, Theorem 1 will show the equilibrium solution of Problem (11).

Theorem 1. *If $\rho_m \leqslant F_1(\rho_1, \rho_2)$, there is no equilibrium solution to problem (11); if $F_1(\rho_1, \rho_2) < \rho_m < F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$, the manufacturer's profit Π_M achieves its maximum at $(\hat{t}^{ct^*}, \hat{m}^{ct^*}) = (0, m_0(0))$; Otherwise, the manufacturer's profit Π_M achieves its maximum at $(\hat{t}^{ct^*}, \hat{m}^{ct^*}) = (t_0, m_0(t_0))$.*[2]

Proof. See Appendix [3]. □

[2] Theorem 1 can be explained intuitively as below. For any given marginal profits of two retailers, if the manufacturer's marginal profit is smaller than some threshold value (here, $F_1(\rho_1, \rho_2)$), she will not gain any positive profit from the business with the two retailers. After her marginal profit goes up to the threshold value but before reaches another larger threshold value (here, $F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$), she can get positive profits by investing in brand name advertising to support the two retailers' local advertising. If her marginal profit exceeds the larger threshold value, she would obtain more profits from investing in brand name advertising and subsidizing the two retailers' local advertising.

[3] Without consideration of the constraints, Hariga (1996) pointed out that if the solution to the first order optimality conditions is unique and the associated Hessian matrix at such solution is positive (negative) definite, then the first order conditions are the necessary and sufficient conditions for a global minimum (maximum) (see pages 1223, 1242 and 1243 in Hariga, 1996). Moreover, Hariga (1996) stated that the objective function satisfying the above conditions is a convex (concave) function (see the Corollary in page 1243 in Hariga, 1996). Other literatures, such as Teng, Chern, and Yang (1997) (page 795), Zhou, Lau, and Yang (2003) (page 1760, Theorem 1), also clearly indicated that the first-order condition will guarantee an optimal solution if the conditions above mentioned (i.e., the solution to the first order optimality conditions is unique and the associated Hessian matrix at such solution is positive (negative) definite) are met.

Theorem 1 indicates that there might be the equilibrium solution to problem (11) only if $\rho_m > F_1(\rho_1, \rho_2)$. In this case, $\hat{t}^{ct^*}$ and $\hat{m}^{ct^*}$ given by Theorem 1 are the equilibrium shared fraction of local advertising expense and equilibrium brand name investment of the manufacturer in the Stackelberg–Cournot setting. Moreover, the manufacturer would offer the retailers positive local advertising compensation when $\rho_m \geqslant F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$ but nothing when $F_1(\rho_1, \rho_2) < \rho_m < F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$. Substituting $\hat{t}^{ct^*}$ and $\hat{m}^{ct^*}$ into (9) and (10) yields the duopolistic retailers' equilibrium local advertising expenditures $\hat{r}_1^{ct^*}$ and $\hat{r}_2^{ct^*}$, respectively. Then, the duopolistic retailers' maximum profits, $\widehat{\Pi}_{ri}^{ct^*}(i = 1, 2)$, the manufacturer's maximum profit, $\widehat{\Pi}_M^{ct^*}$, and the system's profit, $\widehat{\Pi}_S^{ct^*}$, will be obtained by substituting $(\hat{r}_1^{ct^*}, \hat{r}_2^{ct^*}, \hat{t}^{ct^*}, \hat{m}^{ct^*})$, the unique equilibrium point of the game, into (2)–(5).

From Theorem 1, we have the following corollary.

Corollary 1. *In the Stackelberg–Cournot setting, if $\rho_m \geqslant F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$, the higher the marginal profit of the manufacturer, the more local advertising allowance he is willing to pay for the retailers.*

Proof. If $\rho_m \geqslant F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$, from Theorem 1, we have $\hat{t}^{ct^*} = t_0$. Due to (12), one can easily derive that $\partial t_0/\partial \rho_m > 0$, which gives the proof of Corollary 1. □

4.1.2. Stackelberg–Collusion solution

In this subsection, we assume that the duopolistic retailers recognize their interdependence and agree to act together to maximize the total profit of the downstream retail market. As analyzed in Section 3, the total profit of the downstream retail market is

$$\begin{aligned}\Pi_r &= \Pi_{r1} + \Pi_{r2} \\ &= \rho_1\left[\alpha_1 - \beta r_1^{-u} r_2^{v}(1+m)^{-\delta}\right] + \rho_2\left[\alpha_2 - \beta r_2^{-u} r_1^{v}(1+m)^{-\delta}\right] \\ &\quad - (1-t)(r_1 + r_2) \qquad (14)\end{aligned}$$

The first-order conditions of Π_r with respect to r_1 and r_2 yield

$$\partial\Pi_r/\partial r_1 = (\rho_1 u r_1^{-u-1} r_2^{v} - \rho_2 v r_2^{-u} r_1^{v-1})\beta(1+m)^{-\delta} - (1-t) = 0 \quad (15)$$

$$\partial\Pi_r/\partial r_2 = (-\rho_1 v r_1^{-u} r_2^{v-1} + \rho_2 u r_2^{-u-1} r_1^{v})\beta(1+m)^{-\delta} - (1-t) = 0 \quad (16)$$

From (15) and (16), one can easily derive

$$(r_2/r_1)^{u+v}[u(r_2/r_1) + v] = (\rho_2/\rho_1)[v(r_2/r_1) + u] \quad (17)$$

Let $r_2/r_1 = x$, then (17) can be rewritten as

$$ux^{u+v+1} + vx^{u+v} - (\rho_2/\rho_1)vx - u(\rho_2/\rho_1) = 0 \quad (18)$$

Lemma 1. *There exists a unique positive root Ψ to Eq. (18).*

Proof. See Appendix. □

From Lemma 1, one has

$$r_2 = \Psi \cdot r_1 \quad (19)$$

Substituting (19) into (15) or (16), one can obtain

$$\hat{r}_1^{cn} = (G\beta)^{1/(u-v+1)}(1+m)^{-\delta/(u-v+1)}(1-t)^{-1/(u-v+1)} \quad (20)$$

$$\hat{r}_2^{cn} = \Psi \cdot \hat{r}_1^{cn} \quad (21)$$

where $G = \rho_1 u(\Psi)^v - \rho_2 v(\Psi)^{-u}$.

In the following, we will point out that $\hat{r}_1^{cn}$ and $\hat{r}_2^{cn}$ are actually the equilibrium local advertising policies of the two retailers. Before verifying it, we introduce Lemma 2.

Lemma 2. *$(v\Psi + u)/(u\Psi + v) < u/v$*

Proof. See Appendix. □

Based on Lemma 2, one can obtain the following result.

Theorem 2. *For any given m and t, the total profit of the duopolistic retailers Π_r reaches its maximum at $(\hat{r}_1^{cn}, \hat{r}_2^{cn})$.*

Proof. See Appendix. □

From Theorem 2, we know that $\hat{r}_1^{cn}$ and $\hat{r}_2^{cn}$, given by (20) and (21), are the actual reaction functions of the duopolistic retailers. Since the manufacturer, as the leader, knows the retailers' reaction functions for any m and t he sets, the manufacturer's profit maximization problem can be formulated as (obtained by substituting (20) and (21) into (4))

$$\begin{aligned}\max_{m,t} \Pi_M &= \rho_m(\alpha_1 + \alpha_2) - [\rho_m\beta(\Psi^v + \Psi^{-u}) + (1+\Psi)G\beta t/(1-t)] \\ &\quad \times (G\beta)^{(-u+v)/(u-v+1)}(1+m)^{-\delta/(u-v+1)}(1-t)^{(u-v)/(u-v+1)} - m \\ \text{s.t.} &\quad 0 \leqslant t < 1,\ m \geqslant 0. \end{aligned} \quad (22)$$

After neglecting the constraints and from the first-order optimality condition of Π_M with respect to m and t, one can obtain the unique root (t_1, m_1) as follows:

$$t_1 = 1 - (1+\Psi)/\{[(\Psi^v + \Psi^{-u})\rho_m/G - (1+\Psi)](u-v)\} \quad (23)$$

$$\begin{aligned}m_1(t_1) = [&(G\beta)^{1/(u-v+1)}(1+\Psi)(1-t_1)^{-1/(u-v+1)} \\ &\delta/(u-v)]^{(u-v+1)/(\delta+u-v+1)} - 1 \end{aligned} \quad (24)$$

If considering the constraints $0 \leqslant t < 1$ and $m \geqslant 0$ and defining $F_2(\rho_1, \rho_2) = (1 + \Psi)G/(\Psi^v + \Psi^{-u})$, one has the following theorem.

Theorem 3. *If $\rho_m \leqslant F_2(\rho_1, \rho_2)$, there exists no equilibrium solution to Problem (22); if $F_2(\rho_1, \rho_2) < \rho_m < F_2(\rho_1, \rho_2)[1 + 1/(u - v)]$, the manufacturer's profit Π_M achieves its maximum at $(\hat{t}^{cn^*}, \hat{m}^{cn^*}) = (0, m_1(0))$; Otherwise, the manufacturer's profit Π_M achieves its maximum at $(\hat{t}^{cn^*}, \hat{m}^{cn^*}) = (t_1, m_1(t_1))$.*

Through a similar method as that used in the proof of Theorem 1, the proof of Theorem 3 will be easily obtained and, hence, is omitted in this paper.

Theorem 3 shows that there is an equilibrium solution between the manufacturer and the duopolistic retailers in the Stackelberg–Collusion setting only if $\rho_m > F_2(\rho_1, \rho_2)$, and that $\hat{t}^{cn^*}$ and $\hat{m}^{cn^*}$, given by Theorem 3, are the equilibrium shared fraction of local advertising expense and brand name investment of the manufacturer in this setting. Similar to the Stackelberg–Cournot setting, the manufacturer would offer the retailers positive local advertising compensation when $\rho_m \geqslant F_2(\rho_1, \rho_2)[1 + 1/(u - v)]$ but nothing when $F_2(\rho_1, \rho_2) < \rho_m < F_2(\rho_1, \rho_2)[1 + 1/(u - v)]$. Substituting $(\hat{t}^{cn^*}, \hat{m}^{cn^*})$ into (20) and (21) yields the duopolistic retailers' equilibrium local advertising expenditures $\hat{r}_i^{cn^*}$ and $\hat{r}_2^{cn^*}$, respectively. Then, through (2)–(5) one will give the duopolistic retailers' maximum profits, $\widehat{\Pi}_{ri}^{cn^*}$ $(i = 1, 2)$, the manufacturer's maximum profit, $\widehat{\Pi}_M^{cn^*}$, and the system's profit, $\widehat{\Pi}_S^{cn^*}$.

In addition, from (20), (21), (23) and (24), one easily observes that the results of Property 1 and Corollaries 1 and 3 presented in the previous subsection still hold in the Stackelberg–Collusion setting. They are omitted in this subsection for brevity.

4.2. The Vertical Nash game between two echelons in the supply chain

In Section 4.1, we focused on the Stackelberg game structure between two echelons, in which the manufacturer as the leader

Table 1
The equilibrium solutions in Nash-• settings.

	Cournot	Collusion
t	0	0
m	$[\rho_m\beta\delta(\Phi^v+\Phi^{-u})]^{(u-v+1)/(\delta+u-v+1)}(H\beta)^{(-u+v)/(\delta+u-v+1)}-1$	$[\rho_m\beta\delta(\Psi^v+\Psi^{-u})]^{(u-v+1)/(\delta+u-v+1)}(G\beta)^{(-u+v)/(\delta+u-v+1)}-1$
r_1	$[\rho_m\beta\delta(\Phi^v+\Phi^{-u})]^{-\delta/(\delta+u-v+1)}(H\beta)^{(\delta+1)/(\delta+u-v+1)}$	$[\rho_m\beta\delta(\Psi^v+\Psi^{-u})]^{-\delta/(\delta+u-v+1)}(G\beta)^{(\delta+1)/(\delta+u-v+1)}$
r_2	$\Phi\cdot[\rho_m\beta\delta(\Phi^v+\Phi^{-u})]^{-\delta/(\delta+u-v+1)}(H\beta)^{(\delta+1)/(\delta+u-v+1)}$	$\Psi\cdot[\rho_m\beta\delta(\Psi^v+\Psi^{-u})]^{-\delta/(\delta+u-v+1)}(G\beta)^{(\delta+1)/(\delta+u-v+1)}$

holds extreme power and has complete control over the duopolistic retailers, and the two retailers as the follower are presumably powerless to influence the manufacturer.[4] In this subsection, we relax the leader–follower relationship and assume a symmetric one between the manufacturer and the duopolistic retailers. It is assumed that the manufacturer in the upstream market and the duopolistic retailers in the downstream market simultaneously and non-cooperatively maximize their profits. This simultaneous move game is referred to as Vertical Nash game. Following Section 4.1, we still consider the duopolistic retailers' two different behaviors: Cournot and Collusion.

4.2.1. Nash–Cournot solution

Consider now the situation where two echelons in the supply chain play Vertical-Nash game while two competing retailers in the downstream market adopt the Cournot strategy. Then, in this Nash–Cournot setting, all three members in the system will determine their own optimal advertising policies simultaneously. Hence, the manufacturer's optimization problem is

$$\begin{aligned}\max_{m,t} \Pi_M = {} & \rho_m\left[(\alpha_1+\alpha_2)-\beta\left(r_1^{-u}r_2^{v}+r_2^{-u}r_1^{v}\right)(1+m)^{-\delta}\right] \\ & -t(r_1+r_2)-m \\ \text{s.t.} \quad & 0\leqslant t<1,\ \ m\geqslant 0.\end{aligned} \tag{25}$$

and the duopolistic retailers' are

$$\max_{r_1\geqslant 0} \Pi_{r1} = \rho_1[\alpha_1-\beta r_1^{-u}r_2^{v}(1+m)^{-\delta}]-(1-t)r_1 \tag{26}$$

$$\max_{r_2\geqslant 0} \Pi_{r2} = \rho_2[\alpha_2-\beta r_2^{-u}r_1^{v}(1+m)^{-\delta}]-(1-t)r_2 \tag{27}$$

By using the similar method that presented in Section 4.1, one can easily derive the unique equilibrium advertising scheme in the Nash–Cournot setting. The result is listed in Table 1.

From Table 1, we can obtain the following property of the equilibrium solution under the Nash–Cournot game structure.

Property 2. *(i) The manufacturer would not share any portion of local advertising costs with the duopolistic retailers in the Nash–Cournot setting. (ii) The local advertising costs that the duopolistic retailers pay would decrease as the level of brand name investment of the manufacturer increases. (iii) A greater investment of one retailer in local advertising would lead to the other retailer's more spending on local advertising. (iv) The higher the marginal profit of the manufacturer is, the more he will spend on brand name investment and the less the retailers will invest in the local advertising.*

As illustrated by Property 2 (i), the best strategy for the manufacturer in the Nash–Cournot setting, regardless of how the retailers set their local advertising costs, is not to offer any advertising allowance. This is confirmed by the fact that Hisense Electric Co., Ltd., one of the largest LCD TV makers in China, does not give any advertising allowance to its retailers, Wal-Mart and Carrefour. The other results in Property 2 remain the same as those in Property 1.

Similarly, substituting equilibrium solution presented in Table 1 into (2)–(5) yields the duopolistic retailers' maximum profits, $\overline{\Pi}_{ri}^{ct^*}$ $(i=1,2)$, the manufacturer's maximum profit, $\overline{\Pi}_{M}^{ct^*}$, and the system's profit, $\overline{\Pi}_{S}^{ct^*}$, respectively.

4.2.2. Nash–Collusion solution

As discussed in Section 4.1.2, when the duopolistic retailers pursue the collusion solution, their total profit would be that of (14) with their optimal reaction functions in the form of (20) and (21), respectively, for any given t and m. Analogous to Section 4.1, one can easily obtain the unique equilibrium advertising policies in the Nash–Collusion setting which are listed in Table 1. The derivation of equilibrium solution in the Nash–Collusion setting can be seen in Appendix.

Substituting the equilibrium solution into (2)–(5) gives the duopolistic retailers' maximum profits, $\overline{\Pi}_{ri}^{cn^*}$ $(i=1,2)$, the manufacturer's maximum profit, $\overline{\Pi}_{M}^{cn^*}$, and the system's profit, $\overline{\Pi}_{S}^{cn^*}$, respectively.

Additionally, it should be pointed out that the results of Property 2 are still true in the Nash–Collusion case. They are also omitted here for brevity.

4.3. Comparison of optimal solutions among the four settings

4.3.1. A special case with $\alpha_1 = \alpha_2 = \alpha$ and $\rho_1 = \rho_2 = \rho$

We now compare the equilibrium solutions of the four settings presented above. For convenience, we first consider a special case with $\alpha_1 = \alpha_2 = \alpha$ and $\rho_1 = \rho_2 = \rho$. This case can be intuitively explained as that two retailers face the similar market demand and have the same marginal profit. Using the similar analyses as in the previous subsections, we can derive straightforwardly the equilibrium solutions for the four settings in this special case. Results are given in Tables 2–5. Tables 2–4 present the equilibrium solutions for the Stackelberg–Cournot and Stackelberg–Collusion in the special case under three conditions $\rho_m \geqslant \rho u[1+1/(u-v)]$, $\rho u < \rho_m < \rho_m u/(u-v) < \rho u\ [1+1/(u-v)]$, and $\max\{\rho u, \rho(u-v)[1+1/(u-v)]\} < \rho_m\ \ < \rho u[1+1/(u-v)]$, which respectively represent that the manufacturer gives positive local advertising allowance to the two retailers in both settings mentioned above (i.e. $0 < t_0 < 1$, $0 < t_1 < 1$), that the manufacturer does not share any portion of local advertising costs with retailers in the two settings (i.e. $t_0 < 0$, $t_1 < 0$), and that the manufacturer does not give any local advertising allowance to retailers in the Stackelbert-Cournot setting but offers positive local advertising allowance to the retailers in the Stackelbert-Collusion setting (i.e. $t_0 < 0$, $0 < t_1 < 1$). Table 5 shows the equilibrium solutions for Nash–Cournot and Nash–Collusion settings in the special case.

From Tables 2–5, we derive the following Theorems 4–7. Theorem 4 compares the equilibrium advertising policies and corresponding profits of all participants in the Stackelberg–Cournot setting to those in the Stackelberg–Collusion setting, while Theorem 5 shows a comparison of all partners' equilibrium advertising policies and profits between in Nash–Cournot and Nash–Collusion

[4] In general, there are two classification methods for the game structure. One is from the viewpoint of the decision sequence; the other is from the viewpoint of the power balance between the manufacturer and the retailer. For example, according to the different power balance scenarios, Choi (1991) classified the game between the manufacturer and the retailer into three types: Manufacturer-Stackelberg (manufacturer is the leader in the Stackelberg game), Vertical Nash (simultaneous move game), and Retailer-Stackelberg (retailer is the leader in the Stackelberg game).

Table 2
The equilibrium solutions for the special case with $\rho_m/\rho u \geqslant [1 + 1/(u - v)]$ in Stackelberg–• settings.

	Cournot	Collusion
t	$1-\frac{1}{[\rho_m/(\rho u)-1](u-v)}$	$1-\frac{1}{\{\rho_m/[\rho(u-v)]-1\}(u-v)}$
m	$\left\{\frac{2\delta}{u-v}[(\rho_m-\rho u)(u-v)\beta]^{1/(u-v+1)}\right\}^{(u-v+1)/(\delta+u-v+1)}-1$	$\left\{\frac{2\delta}{u-v}[(\rho_m-\rho(u-v))(u-v)\beta]^{1/(u-v+1)}\right\}^{(u-v+1)/(\delta+u-v+1)}-1$
r_i	$[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}[(\rho_m-\rho u)(u-v)\beta]^{1/(\delta+u-v+1)}$	$[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\{[\rho_m-\rho(u-v)](u-v)\beta\}^{1/(\delta+u-v+1)}$
Π_{ri}	$\rho\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}[(\rho_m-\rho u)(u-v)\beta]^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\rho\beta(1+u)$	$\rho\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}\{[\rho_m-\rho(u-v)](u-v)\beta\}^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\cdot\rho\beta(1+u-v)$
Π_M	$2\rho_m\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}[(u-v)\beta]^{\frac{-(\delta+u-v)}{\delta+u-v+1}}(\rho_m-\rho u)^{\frac{1}{\delta+u-v+1}}$ $\cdot 2\beta(\delta+u-v+1)+1$	$2\rho_m\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}[(u-v)\beta]^{\frac{-(\delta+u-v)}{\delta+u-v+1}}[\rho_m-\rho(u-v)]^{\frac{1}{\delta+u-v+1}}\cdot 2\beta(\delta+u-v+1)+1$
Π_S	$2(\rho_m+\rho)\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}[(u-v)\beta(\rho_m-\rho u)]^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\cdot 2\beta[\rho(1+u)$ $+(\rho_m-\rho u)(\delta+u-v+1)]+1$	$2(\rho_m+\rho)\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}\{(u-v)\beta[\rho_m-\rho(u-v)]\}^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\cdot 2\beta\{\rho(1+u-v)+[\rho_m$ $-\rho(u-v)](\delta+u-v+1)\}+1$

Table 3
The equilibrium solutions for the special case with $1 < \rho_m/\rho u < \rho_m/[\rho(u\text{-}v)] < [1 + 1/(u - v)]$ in Stackelberg-• settings.

	Cournot	Collusion
t	0	0
m	$(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{(u-v+1)/(\delta+u-v+1)}-1$	$[\rho(u-v)\beta]^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{(u-v+1)/(\delta+u-v+1)}-1$
r_i	$(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}$	$[\rho(u-v)\beta]^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}$
Π_{ri}	$\rho\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}(1+1/u)$	$\rho\alpha-[\rho(u-v)\beta]^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}[1+1/(u-v)]$
Π_M	$2\rho_m\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[\rho_m/(\rho u)+\delta/(u-v)]+1$	$2\rho_m\alpha-[\rho(u-v)\beta]^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[\rho_m/(\rho(u-v))+\delta/(u-v)]+1$
Π_S	$2(\rho_m+\rho)\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[(\rho_m+\rho)/$ $(\rho u)+1+\delta/(u-v)]+1$	$2(\rho_m+\rho)\alpha-[\rho(u-v)\beta]^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[(\rho_m+\rho)/$ $(\rho(u-v))+1+\delta/(u-v)]+1$

Table 4
The equilibrium solutions for the special case with $1 < \rho_m/\rho u < [1 + 1/(u - v)]$ & $\rho_m/[\rho(u - v)]. \geqslant [1 + 1/(u - v)]$ in Stackelberg-• settings.

	Cournot	Collusion
t	0	$1-\frac{1}{\{\rho_m/[\rho(u-v)]-1\}(u-v)}$
m	$(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{(u-v+1)/(\delta+u-v+1)}-1$	$\left\{\frac{2\delta}{u-v}[(\rho_m-\rho(u-v))(u-v)\beta]^{1/(u-v+1)}\right\}^{(u-v+1)/(\delta+u-v+1)}-1$
r_i	$(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}$	$[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\{[\rho_m-\rho(u-v)](u-v)\beta\}^{1/(\delta+u-v+1)}$
Π_{ri}	$\rho\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}$ $(1+1/u)$	$\rho\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}\{[\rho_m-\rho(u-v)](u-v)\beta\}^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\cdot\rho\beta(1+u-v)$
Π_M	$2\rho_m\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[\rho_m/(\rho u)+\delta/(u-v)]+1$	$2\rho_m\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}[(u-v)\beta]^{\frac{-(\delta+u-v)}{\delta+u-v+1}}[\rho_m-\rho(u-v)]^{\frac{1}{\delta+u-v+1}}\cdot 2\beta(\delta+u-v+1)+1$
Π_S	$2(\rho_m+\rho)\alpha-(\rho u\beta)^{1/(\delta+u-v+1)}[2\delta/(u-v)]^{-\delta/(\delta+u-v+1)}\cdot 2[(\rho_m+\rho)/(\rho u)+1+\delta/(u-v)]+1$	$2(\rho_m+\rho)\alpha-\left(\frac{2\delta}{u-v}\right)^{\frac{-\delta}{\delta+u-v+1}}\{(u-v)\beta[\rho_m-\rho(u-v)]\}^{\frac{-(\delta+u-v)}{\delta+u-v+1}}\cdot 2\beta\{\rho(1+u-v)+[\rho_m-\rho(u-v)](\delta+u-v+1)\}+1$

Table 5
The equilibrium solutions for the special case in Nash-• settings.

	Cournot	Collusion
t	0	0
m	$(2\rho_m\beta\delta)^{(u-v+1)/(\delta+u-v+1)}(\rho\beta u)^{(-u+v)/(\delta+u-v+1)}-1$	$(2\rho_m\beta\delta)^{(u-v+1)/(\delta+u-v+1)}[\rho\beta(u-v)]^{(-u+v)/(\delta+u-v+1)}-1$
r_i	$(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}(\rho\beta u)^{(\delta+1)/(\delta+u-v+1)}$	$(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}[\rho\beta(u-v)]^{(\delta+1)/(\delta+u-v+1)}$
Π_{ri}	$\rho\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}(\rho\beta u)^{(-u+v)/(\delta+u-v+1)}\rho\beta(1+u)$	$\rho\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}[\rho\beta(u-v)]^{(-u+v)/(\delta+u-v+1)}\rho\beta(1+u-v)$
Π_M	$2\rho_m\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}(\rho\beta u)^{(-u+v)/(\delta+u-v+1)}2\rho_m\beta(1+\delta)+1$	$2\rho_m\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}[\rho\beta(u-v)]^{(-u+v)/(\delta+u-v+1)}2\rho_m\beta(1+\delta)+1$
Π_S	$2(\rho_m+\rho)\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}(\rho\beta u)^{(-u+v)/(\delta+u-v+1)}\cdot 2\beta[\rho(1+u)+\rho_m(1+\delta)]+1$	$2(\rho_m+\rho)\alpha-(2\rho_m\beta\delta)^{-\delta/(\delta+u-v+1)}[\rho\beta(u-v)]^{(-u+v)/(\delta+u-v+1)}\cdot 2\beta[\rho(1+u-v)+\rho_m(1+\delta)]+1$

settings. Theorem 6 (Theorem 7) presents the influence of different game structures, Stackelberg and Vertical Nash, between the two echelons on all partners' advertising policies and profits, assuming the duopolistic retailers to choose Cournot (Collusion).

Theorem 4.

(1) *If $\rho_m \geqslant \rho u\ [1 + 1/(u - v)]$, i.e., the manufacturer's marginal profit considerably exceeds the retailers', then*
(i) $\hat{t}^{ct^*} < \hat{t}^{cn^*}, \hat{m}^{ct^*} < \hat{m}^{cn^*}$ *and* $\hat{r}_i^{ct^*} < \hat{r}_i^{cn^*}$; *(ii)* $\widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*}, \widehat{\Pi}_M^{ct^*} > \widehat{\Pi}_M^{cn^*}$ *and* $\widehat{\Pi}_S^{ct^*} < \widehat{\Pi}_S^{cn^*}$.

(2) *If $\rho u < \rho_m < \rho_m u/(u\text{-}v) < \rho u\ [1 + 1/(u - v)]$, i.e., the manufacturer's marginal profit has no significant difference with the retailers', then*
(i) $\hat{t}^{ct^*} = \hat{t}^{cn^*} = 0, \hat{m}^{ct^*} > \hat{m}^{cn^*}$ *and* $\hat{r}_i^{ct^*} > \hat{r}_i^{cn^*}$;
(ii)

$$\begin{cases} \widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*} & \eta_1(u-v) > \eta_1(u) \\ \widehat{\Pi}_{ri}^{ct^*} \geqslant \widehat{\Pi}_{ri}^{cn^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{M}^{ct^*} < \widehat{\Pi}_{M}^{cn^*} & \eta_2(u-v) > \eta_2(u) \\ \widehat{\Pi}_{M}^{ct^*} \geqslant \widehat{\Pi}_{M}^{cn^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{S}^{ct^*} < \widehat{\Pi}_{S}^{cn^*} & \eta_1(u-v)+\eta_2(u-v) > \eta_1(u)\eta_2(u) \\ \widehat{\Pi}_{S}^{ct^*} \geqslant \widehat{\Pi}_{S}^{cn^*} & \text{Otherwise}. \end{cases}$$

where $\eta_1(x)=-x^{1/(\delta+u-v+1)}(1+1/x)$, $\eta_2(x)=-x^{1/(\delta+u-v+1)}[\delta/(u\text{-}v)+\rho_m/(\rho x)]$.

(3) *If* $\max\{\rho u, \ \rho(u\text{-}v)[1 + 1/(u\text{-}v)]\} < \rho_m < \rho u[1 + 1/(u - v)]$, *i.e., the manufacturer's marginal profit is comparatively higher than the retailers', then*

(i)

$$\hat{t}^{ct^*} = 0 < \hat{t}^{cn^*}, \quad \begin{cases} \hat{m}^{ct^*} < \hat{m}^{cn^*} & \omega < 1 \\ \hat{m}^{ct^*} \geqslant \hat{m}^{cn^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \hat{r}_i^{ct^*} < \hat{r}_i^{cn^*} & \omega < 1 \\ \hat{r}_i^{ct^*} \geqslant \hat{r}_i^{cn^*} & \text{Otherwise}, \end{cases}$$

where $\omega = \rho u/\{[\rho_m\text{-}\rho(u\text{-}v)](u\text{-}v)\}$.

(ii)

$$\begin{cases} \widehat{\Pi}_{ri}^{ct^*} > \widehat{\Pi}_{ri}^{cn^*} & \omega^{\delta+u-v} > [(u+1)/(u+1-v)]^{(\delta+u-v+1)} \\ \widehat{\Pi}_{ri}^{ct^*} \leqslant \widehat{\Pi}_{ri}^{cn^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{M}^{ct^*} > \widehat{\Pi}_{M}^{cn^*} & \omega^{-1/(\delta+u-v+1)} > [\rho_m(u-v) + \delta\rho u] \\ & \qquad /[\rho u(\delta + u - v + 1)] \\ \widehat{\Pi}_{M}^{ct^*} \leqslant \widehat{\Pi}_{M}^{cn^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{S}^{ct^*} > \widehat{\Pi}_{S}^{cn^*} & \omega^{(\delta+u-v)/(\delta+u-v+1)} < \{\rho(1+u-v) + [\rho_m - \rho(u-v)](\delta+u-v+1)\} \\ & \qquad /\{\rho_m + \rho[1+u+\delta u/(u-v)]\} \\ \widehat{\Pi}_{S}^{ct^*} > \widehat{\Pi}_{S}^{cn^*} & \text{Otherwise}. \end{cases}$$

From Theorem 4, one can observe the following. If the manufacturer gives positive advertising allowance to the two retailers in both Stackelberg–Cournot and Stackelberg–Collusion settings: (i) The retailers would spend more money on local advertising under Collusion than did under Cournot, while the manufacturer would pay more brand name investment and more local advertising allowance under the retailers' Collusion than under the retailers' Cournot. That is to say, if the two retailers are followers in determining advertising investment, their collusion would lead to the increase of their local advertising investment, which is out of our expectation. The intuitive interpretation of this observation is that when the retailers lack power in determining advertising investment, they can increase their own local advertising investment through colluding to encourage the manufacturer enhancing brand name investment and local advertising allowance. This observation can be confirmed by the fact that GOME has ever colluded with Suning to demand the higher local advertising allowance from Gree. (ii) The manufacturer gains more profits under two retailers' Cournot strategy than under two retailers' Collusion strategy. It is just contrary for the two retailers. Hence, the manufacturer always prefers the retailers to adopt Cournot strategy rather than act in Collusion, whereas the duopolistic retailers always have a preference for Collusion. This phenomenon is very common in practice. For example, Gree always hopes that GOME and Suning play Cournot game, while GOME and Suning have an incentive to collude. From the perspective of the supply chain system, however, the duopolistic retailers' Collusion strategy is superior to their Cournot strategy. It implies that being a follower in the Stackelberg game, the retailers choosing Collusion will not only benefit themselves but also benefit the whole channel, as compared to Cournot strategy. However, if the manufacturer does not offer the two retailers positive advertising allowance in both Stackelberg–Cournot and Stackelberg–Collusion settings or provides the retailers with positive advertising allowance only in one of the two settings, the characteristics of the equilibrium advertising policies and preferences in two retailers' different behaviors of all participants mentioned above would not hold but vary with changes of parameters' values.

Theorem 5. *(i)* $\bar{t}^{ct^*} = \bar{t}^{cn^*} = 0, \bar{m}^{ct^*} < \bar{m}^{cn^*}, \bar{r}_i^{ct^*} > \bar{r}_i^{cn^*}$; *(ii)* $\overline{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{cn^*}, \overline{\Pi}_{M}^{ct^*} > \overline{\Pi}_{M}^{cn^*}, \overline{\Pi}_{S}^{ct^*} > \overline{\Pi}_{S}^{cn^*}$.

Theorem 5 shows that when the two echelons implement Vertical-Nash game, the two retailers in the downstream market would invest much less in local advertising under Collusion than did under Cournot, whereas the manufacturer in the upstream would pay more brand name investment if the retailers choose Collusion than did if the retailers adopt Cournot. This implies that through collusion the two retailers reduce their own local advertising payment and force the manufacturer to enhance brand name investment, which is consistent with our expectation.

Theorem 6.

(1) *If* $\rho_m \geqslant \rho u \ [1 + 1/(u - v)]$, *i.e., the manufacturer's marginal profit considerably exceeds the retailers', then*
  (i) *we* $\hat{t}^{ct^*} \geqslant \bar{t}^{ct^*}, \hat{m}^{ct^*} < \bar{m}^{ct^*}, \hat{r}_i^{ct^*} > \bar{r}_i^{ct^*}$;
  (ii) $\widehat{\Pi}_{ri}^{ct^*} > \overline{\Pi}_{ri}^{ct^*}, \widehat{\Pi}_{M}^{ct^*} > \overline{\Pi}_{M}^{ct^*}, \widehat{\Pi}_{S}^{ct^*} > \overline{\Pi}_{S}^{ct^*}$.

(2) *If* $\rho u < \rho_m < \rho_m u/(u\text{-}v) < \rho u \ [1 + 1/(u - v)]$ *or* $\max\{\rho u, \ \rho(u\text{-}v)[1 + 1/(u\text{-}v)]\} < \rho_m < \rho u[1 + 1/(u - v)]$, *i.e., the manufacturer's marginal profit either has no significant difference with the retailers', or is comparatively higher than the retailers', then*

(i)

$$\hat{t}^{ct^*} = \bar{t}^{ct^*}, \begin{cases} \hat{m}^{ct^*} < \bar{m}^{ct^*} & \rho_m(u-v) > \rho u \\ \hat{m}^{ct^*} \geqslant \bar{m}^{ct^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \hat{r}_i^{ct^*} > \bar{r}_i^{ct^*} & \rho_m(u-v) > \rho u \\ \hat{r}_i^{ct^*} \leqslant \bar{r}_i^{cn^*} & \text{Otherwise}; \end{cases}$$

(ii)

$$\begin{cases} \widehat{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{ct^*} & \rho_m(u-v) > \rho u \\ \widehat{\Pi}_{ri}^{ct^*} \geqslant \overline{\Pi}_{ri}^{ct^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{M}^{ct^*} < \overline{\Pi}_{M}^{ct^*} & \rho_m(u-v) > \rho u \\ \widehat{\Pi}_{M}^{ct^*} \geqslant \overline{\Pi}_{M}^{ct^*} & \text{Otherwise}, \end{cases}$$

$$\begin{cases} \widehat{\Pi}_{S}^{ct^*} < \overline{\Pi}_{S}^{ct^*} & \rho_m(u-v) > \rho u \\ \widehat{\Pi}_{S}^{ct^*} \geqslant \overline{\Pi}_{S}^{ct^*} & \text{Otherwise}. \end{cases}$$

Theorem 7.

(1) *If* $\rho_m \geqslant \rho u \ [1 + 1/(u - v)]$ *or* $\max\{\rho u, \ \rho(u\text{-}v)[1 + 1/(u\text{-}v)]\} < \rho_m < \rho u[1 + 1/(u - v)]$, *i.e., the manufacturer's marginal profit either considerably exceeds the retailers' or is comparatively higher than the retailers', thens*
  (i) $\hat{t}^{cn^*} \geqslant \bar{t}^{cn^*}, \hat{m}^{cn^*} < \bar{m}^{cn^*}, \hat{r}_i^{cn^*} > \bar{r}_i^{cn^*}$; *(ii)* $\widehat{\Pi}_{ri}^{cn^*} > \overline{\Pi}_{ri}^{cn^*}, \widehat{\Pi}_{M}^{cn^*} > \overline{\Pi}_{M}^{cn^*}, \widehat{\Pi}_{S}^{cn^*} > \overline{\Pi}_{S}^{cn^*}$.

(2) *If* $\rho u < \rho_m < \rho_m u/(u\text{-}v) < \rho u \ [1 + 1/(u - v)]$, *i.e., the manufacturer's marginal profit has no significant difference with the retailers', then*

(i)

$$\hat{t}^{cn^*} = \bar{t}^{cn^*}, \begin{cases} \hat{m}^{cn^*} < \bar{m}^{cn^*} & \rho_m > \rho \\ \hat{m}^{cn^*} \geqslant \bar{m}^{cn^*} & \text{Otherwise}, \end{cases} \begin{cases} \hat{r}_i^{cn^*} > \bar{r}_i^{cn^*} & \rho_m > \rho \\ \hat{r}_i^{cn^*} \leqslant \bar{r}_i^{cn^*} & \text{Otherwise}; \end{cases}$$

(ii)

$$\begin{cases} \widehat{\Pi}_{ri}^{cn^*} < \overline{\Pi}_{ri}^{cn^*} & \rho_m > \rho \\ \widehat{\Pi}_{ri}^{cn^*} \geqslant \overline{\Pi}_{ri}^{cn^*} & \text{Otherwise}, \end{cases} \begin{cases} \widehat{\Pi}_{M}^{cn^*} < \overline{\Pi}_{M}^{cn^*} & \rho_m > \rho \\ \widehat{\Pi}_{M}^{cn^*} \geqslant \overline{\Pi}_{M}^{cn^*} & \text{Otherwise}, \end{cases} \begin{cases} \widehat{\Pi}_{S}^{cn^*} < \overline{\Pi}_{S}^{cn^*} & \rho_m > \rho \\ \widehat{\Pi}_{S}^{cn^*} \geqslant \overline{\Pi}_{S}^{cn^*} & \text{Otherwise}. \end{cases}$$

(The proofs of Theorems 4–7 are presented in Appendix)

From Theorems 6 and 7, one easily has the following observations. As long as the manufacturer is willing to offer the two retailers positive local advertising allowance in the inter-echelon Stackelberg setting, then (i) whether the duopolistic retailers obey the Cournot behavior or Collusion behavior, the local advertising expenditures of the retailers and the advertising allowance level of the manufacturer are higher than those in the inter-echelon Vertical-Nash setting. But, the brand name investment of the manufacturer is smaller than that in the inter-echelon Vertical-Nash setting; (ii) regardless of Cournot or Collusion chosen by the duopolistic retailers in the downstream market, both the manufacturer and the retailers prefer to play Stackelberg game rather than play Vertical-Nash game. This is just contrary to our intuitive expectation that the retailers should prefer the latter to the leader–follower relationship between the two echelons. An intuitive explanation for the phenomenon is as follows. As compared with the Vertical Nash game, in the Stackelberg game, a greater investment in local advertising will lead to the increased sales volume of the product, thus making the retailers achieve higher profits; at the same time, the manufacturer will give the retailers more advertising allowance to compensate for their losses resulted from the increased advertising investment. Consequently, the retailers prefer to play Stackelberg game rather than play Nash game. This finding can be verified by the milk industry. For example, Mengniu (one of the largest milk manufacturers in China) and its retailers—Beijing Hualian and Suguo (two large supermarkets in China)—played the Stackelberg game where the Mengniu is the leader and Beijing Hualian and Suguo are the followers. If the manufacturer does not offer the two retailers any positive advertising allowance in the inter-echelon Stackelberg setting, no matter what strategy the retailers adopt: Cournot or Collusion, the results mentioned above would not hold but vary with changes of parameters' values.

Based on Theorems 4–7, one can easily derive the following corollary.

Corollary 3. *If $\rho_m \geqslant \rho u[1 + 1/(u - v)]$, i.e., the manufacturer's marginal profit considerably exceeds the retailers', then*

(i) $\bar{r}_i^{cn^*} < \bar{r}_i^{ct^*} < \hat{r}_i^{ct^*} < \hat{r}_i^{cn^*}$; (ii) $\hat{m}^{ct^*} < \hat{m}^{cn^*} < \bar{m}^{cn^*}, \hat{m}^{ct^*} < \bar{m}^{ct^*} < \bar{m}^{cn^*}$;

(iii) $\overline{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{cn^*} < \widehat{\Pi}_{ri}^{cn^*}, \overline{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*}$; (iv) $\overline{\Pi}_{M}^{cn^*} < \widehat{\Pi}_{M}^{cn^*} < \widehat{\Pi}_{M}^{ct^*}, \overline{\Pi}_{M}^{cn^*} < \overline{\Pi}_{M}^{ct^*} < \widehat{\Pi}_{M}^{ct^*}$;

(v) $\overline{\Pi}_{S}^{cn^*} < \overline{\Pi}_{S}^{ct^*} < \widehat{\Pi}_{S}^{ct^*} < \widehat{\Pi}_{S}^{cn^*}$. *From Corollary 3, further insights can be drawn in the special case with $\alpha_1 = \alpha_2 = \alpha$ and $\rho_1 = \rho_2 = \rho$.*

If the manufacturer gives positive advertising allowance to the two retailers in the inter-echelon Stackelberg setting, then among the discussed four settings,

(1) Stackelberg–Collusion makes the duopolistic retailers spend the highest amount of money on local advertising, whereas, Nash–Collusion results in the lowest spending on local advertising.
(2) Nash–Collusion makes the manufacturer bear the highest brand name investment and both retailers invest the lowest local advertising expenses, whereas Stackelberg–Cournot leads to the lowest brand name investment of the manufacturer.
(3) Each retailer enjoys the highest profit under Stackelberg–Collusion, yet obtains the lowest profit under Nash–Cournot.
(4) The manufacturer gains the highest profit under Stackelberg–Cournot, while the lowest under Nash–Collusion.
(5) The total profit of the supply chain system is the highest under Stackelberg–Collusion, yet the lowest under Nash–Collusion.

Table 7
Seven groups of values of parameters considered.

Groups	ρ_m	ρ_1	ρ_2	u	v	δ	β	α_1	α_2
Group 1	80	50	40	1.2	0.3	1.0	5×10^7	6000	4000
Group 2	80	70	50	1.1	0.4	0.8	9×10^7	2000	1000
Group 3	55	30	40	1.0	0.5	1.2	8×10^6	800	600
Group 4	180	50	30	0.7	0.4	0.5	4×10^7	4500	6500
Group 5	220	60	70	0.8	0.4	1.2	6×10^7	3000	2000
Group 6	120	80	20	0.9	0.4	1.2	5×10^7	4000	6000
Group 7	200	50	40	2.0	0.6	2.2	7×10^7	4000	3000

4.3.2. The general case with $\alpha_1 \neq \alpha_2$ or $\rho_1 \neq \rho_2$

Naturally, a question to ask is whether the results obtained in the special case with $\alpha_1 = \alpha_2 = \alpha$ and $\rho_1 = \rho_2 = \rho$ still remain in the general case. In order to answer this question, we now assume $\alpha_1 \neq \alpha_2$ or $\rho_1 \neq \rho_2$. This case can be intuitively interpreted as the duopolistic retailers facing the dissimilar market demand or having different marginal profit. Although it is rather complicated to show closed-form results for this case due to its generality, the following can be derived.

Theorem 8. *If the two echelons play Stackelberg game, the equilibrium fraction of local advertising expense the manufacturer shares when the duopolistic retailers choose Cournot would not exceed the counterpart when the retailers act in Collusion, i.e., $\hat{t}^{ct^*} \leqslant \hat{t}^{cn^*}$.*

Proof. See Appendix. □

However, we are not sure theoretically whether other results, occurred in the special case, still remain in the general case. So we will investigate it through numerical experiments. The experiment is implemented in the following manner. First, for each parameter of the model, we extract randomly a value out of its given interval, which is shown in Table 6. We then compute the equilibrium solution of the problem in the four settings based on this group of extracted values of all parameters. We extract randomly more than 300 groups of values of the parameters in total in the experiment. Our observations below are obtained based on the computational results of all groups. For brevity, we pick arbitrarily seven from all groups, in which the values of parameters are listed in Table 7, to illustrate our observations intuitively. Table 8 shows their corresponding equilibrium solutions. From Table 8, one easily observes that the similar insights with those observed from Theorems 4–7 also remain under the general case.

From Theorems 4–7, Corollary 3 and the above analysis, one easily gets the following property.

Table 6
The ranges of parameters.

Parameters	ρ_m	ρ_1	ρ_2	u	v	δ	β (10^7)	α_1	α_2
Ranges	[20, 300]	[20, 100]	[20,100]	[0.4, 2.5]	[0.2, 1.0]	[0.4, 2.5]	[0.5, 10]	[500, 8000]	[500, 8000]

Table 8
The equilibrium solutions in the four settings.

Two echelons	Two retailers	Groups	r_1	r_2	m	t	Π_{r1}	Π_{r2}	Π_M	Π_S
Stackelberg	Cournot	Group 1	1414	1294	3010	0	297405	157626	792945	1247976
		Group 2	6623	5788	14185	0	127356	38947	210511	376814
		Group 3	628	706	3204	0	22742	22588	71672	117002
		Group 4	113122	88696	336362	0.4100	62911	67910	769092	899913
		Group 5	2090	2241	12994	0.2295	176376	136113	1071843	1384333
		Group 6	1901	1040	7059	0.1787	316704	118196	1184116	1619015
		Group 7	90	85	275	0.4250	199922	119926	1399422	1719270
	Collusion	Group 1	1292	1162	2726	0	297301	157518	792383	1247202
		Group 2	5621	4736	11836	0	126686	38159	208003	372848
		Group 3	493	576	2565	0.0894	22603	22476	71227	116307
		Group 4	127217	87619	358060	0.7748	111641	98669	690982	901292
		Group 5	2174	2404	13738	0.6667	177399	137259	1070232	1384889
		Group 6	2264	985	7801	0.6563	317935	118713	1182445	1619093
		Group 7	95	88	288	0.6764	199947	119950	1399394	1719292
Vertical Nash	Cournot	Group 1	1278	1169	3653	0	297656	157856	792692	1248205
		Group 2	6943	6069	12832	0	126744	38413	211124	376282
		Group 3	697	782	2822	0	22607	22437	71826	116870
		Group 4	69651	54612	413157	0	55845	62370	740526	858742
		Group 5	1480	1588	15632	0	176368	136026	1071340	1383734
		Group 6	1389	760	8863	0	316686	118104	1183749	1618540
		Group 7	45	43	445	0	199914	119919	1399353	1719186
	Collusion	Group 1	1056	950	3997	0	297793	157971	792003	1247768
		Group 2	5091	4290	14605	0	127941	39274	207134	374349
		Group 3	386	452	3215	0	22797	22688	71104	116589
		Group 4	36075	24846	481234	0	82289	73725	536295	692309
		Group 5	810	895	17401	0	177093	136936	1068093	1382122
		Group 6	884	384	10383	0	317655	118538	1180963	1617156
		Group 7	36	33	494	0	199939	119942	1399278	1719159

Property 3. *If the manufacturer's marginal profit considerably exceeds the retailers, no matter which kind of game structure faced by the supply chain members, the retailers have incentives to transit to the Stackelberg–Collusion structure, whereas the manufacturer has motivation to transit to the Stackelberg–Cournot structure. Otherwise, whether the supply chain members have an incentive to transit to a different game structure depends on the values of parameters.*

In addition, we study the influence of variation of ρ_m, ρ_1, ρ_2 and v on the equilibrium solutions for all participants in the previous four settings. Table 9 (Table 10) shows the effect of variation of marginal profits of the retailers (the manufacturer) on the equilibrium advertising policies and corresponding profits for all participants in the previous four settings. Table 11 presents the influence of variation of sensitivity degree of one retailer's sales volume to the other's local advertising investment on the equilibrium solutions in the four settings when the marginal profits of the manufacturer and the retailers are given.

From Tables 9–11, one can obtain the following implications.

(1) In the four settings, the ratio of two retailers' local advertising expenditures and the ratio of their profits are both positively correlated to the ratio of their marginal profits, which means that if one member's marginal profit is higher as compared to the other's, then he should spend more on local advertising, which in turn brings him more profits, than does the other.

(2) In the inter-echelon Stackelberg setting, the higher the marginal profit of the manufacturer, the more the manufacturer invests in brand name advertisement and local advertising allowance, which results in a higher local advertising investment of both retailers. However, in the inter-echelon Nash setting, the higher the marginal profit for the manufacturer, the more he would spend only on brand name investment, consequently, the less the retailers in turn spend on the local advertising. In addition, the profits of all participants increase as the marginal profit of the manufacturer increases in each of the four settings (see Table 9).

Table 9
The effect of variation of ρ_1 and ρ_2 on the equilibrium solutions in the four settings.

Two echelons	Two retailers	ρ_1	ρ_2	ρ_1/ρ_2	r_1/r_2	m	t	Π_{r1}/Π_{r2}	Π_M
Stackelberg	Cournot	50	30	1.6667	1.2754	72110	0.6340	1.0885	1747616
		40	40	1	1	71462	0.6190	0.6270	1749881
		40	50	0.8	0.8992	70776	0.5644	0.4920	1752282
	Collusion	50	30	1.6667	1.3536	74841	0.7888	1.0979	1738055
		40	40	1	1	73849	0.7778	0.6355	1741527
		40	50	0.8	0.8662	73552	0.7490	0.5001	1742568
Vertical Nash	Cournot	50	30	1.6667	1.2754	103291	0	1.0830	1724557
		40	40	1	1	101849	0	0.6180	1728402
		40	50	0.8	0.8992	99300	0	0.4849	1735199
	Collusion	50	30	1.6667	1.3536	116430	0	1.0579	1689520
		40	40	1	1	113908	0	0.6218	1696243
		40	50	0.8	0.8662	111224	0	0.4874	1703402

$u = 0.8$, $v = 0.3$, $\delta = 0.6$, $\beta = 5 \times 10^7$, $\alpha_1 = 4000$, $\alpha_2 = 6000$, $\rho_m = 200$.

Table 10
The effect of variation of ρ_m on the equilibrium solutions in the four settings.

Two echelons	Two retailers	ρ_m	r_1	r_2	m	t	Π_{r1}	Π_{r2}	Π_M	Π_S
Stackelberg	Cournot	40	18798	20905	47644	0	117705	252963	307954	678622
		70	18798	20905	47643	0	117705	252963	574652	945320
		150	23491	26125	59540	0.3738	126902	263191	121610	1681703
		200	27924	31055	70776	0.5644	132634	269566	1752282	2154482
	Collusion	40	14731	17006	38084	0	114566	250223	304997	669786
		70	15208	17557	39318	0.0648	116133	251940	562384	930458
		150	24308	28062	62844	0.6507	133812	271308	1280047	1685167
		200	28450	32843	73552	0.7490	137974	275868	1742568	2156410
Vertical Nash	Cournot	40	22194	24682	31455	0	110064	244465	316121	670649
		70	18914	21035	46912	0	117442	252671	574901	945015
		150	15213	16919	80855	0	125770	261932	1284386	1672088
		200	14013	15584	99300	0	128471	264936	1735199	2128606
	Collusion	40	15197	17544	35232	0	113129	248649	306049	667827
		70	12951	14951	52545	0	120055	256237	559897	936171
		150	10417	12026	90564	0	127871	264800	1258495	1651167
		200	9595	11077	111224	0	130407	267578	1703402	2101387

$u = 0.8$, $v = 0.3$, $\delta = 0.6$, $\beta = 5 \times 10^7$, $\alpha_1 = 4000$, $\alpha_2 = 6000$, $\rho_1 = 40$, $\rho_2 = 50$.

Table 11
The effect of variation of v on the equilibrium solutions in the four settings.

Two echelons	Two retailers	v	r_1	r_2	m	t	Π_{r1}	Π_{r2}	Π_M	Π_S
Stackelberg	Cournot	0.3	33682	26409	72109	0.6340	172265	158254	1747616	2078136
		0.4	46950	37222	126260	0.5433	151754	141751	1579133	1872639
		0.5	64252	51455	231415	0.3920	112101	109607	1267185	1488893
		0.6	82070	66336	445219	0.0892	31822	44064	664343	740230
	Collusion	0.3	36272	26095	74841	0.7888	178188	162292	1738055	2078535
		0.4	51605	36962	132852	0.7942	165345	150869	1557160	1873374
		0.5	71994	51544	247077	0.7993	142647	129908	1217590	1490146
		0.6	93689	67263	482860	0.8040	100920	89799	551418	742138
Vertical Nash	Cournot	0.3	14925	11702	103291	0	166417	153669	1724557	2044643
		0.4	23850	18908	166091	0	146337	137457	1557090	1840884
		0.5	40081	32098	280736	0	109817	107778	1251370	1468966
		0.6	71428	57734	502960	0	39285	50097	658774	748156
	Collusion	0.3	10780	7755	116430	0	169309	155083	1689520	2013913
		0.4	14226	10189	192636	0	153572	140973	1486303	1780847
		0.5	18279	13087	331381	0	127446	116632	1116317	1360396
		0.6	21727	15599	593735	0	82765	73271	416706	572742

$u = 0.8$, $\delta = 0.6$, $\beta = 5 \times 10^7$, $\alpha_1 = 4000$, $\alpha_2 = 6000$, $\rho_m = 200$, $\rho_1 = 50$, $\rho_2 = 30$.

(3) In each of the four settings, the bigger the value of v, the higher the local advertising expense of each retailer and the brand name investment of the manufacturer but the lower the profits of the manufacturer, two retailers and the supply chain system. The reason why this happens is as follows. Since the demand each retailer faces is negatively correlated to the substitutability degree v, both retailers' demand will decrease as v increases, which directly results in a decrease of the manufacturer's profit. Thus, from the manufacturer's viewpoint, enhancing the brand name investment to increase the demand for both retailers would be a necessary way to improve his profit. On the other hand, since the size of demand each retailer faces is dependent on the amount of money that the two retailers spend on local adverting, a greater local advertising investment of one retailer would lead to a higher demand of himself but a lower demand of the other. Therefore, each retailer will increase its local advertising expenditures so as to grab the more market demand as v increases. Additionally, in the inter-echelon Stackelberg setting, as v increases, the fraction of local advertising expenditure t shared by the manufacturer decreases in the Stackelberg–Cournot setting but increases in the Stackelberg–Collusion setting. It implies that the manufacturer could reduce the loss of profit brought by the competition of two retailers in the Stackelberg–Cournot setting by decreasing the local advertising allowance for two retailers. However, in the Stackelberg–Collusion setting, through collusion the two retailers would force the manufacturer to increase rather than to decrease the local advertising allowance as two retailers' local advertising expenditures increase (see Table 10).

5. The centralized decision model

Consider now a situation where both the manufacturer and the duopolistic retailers are willing to cooperate to pursue the centralized optimal advertising policies. Hence, unlike in the decentralized case, the objective in this setting is to maximize the total profit of the system. That is

$$\max_{m,r_1,r_2} \Pi_S = (\rho_1+\rho_m)[\alpha_1 - \beta r_1^{-u} r_2^{v}(1+m)^{-\delta}](\rho_2+\rho_m)[\alpha_2 - \beta r_2^{-u} r_1^{v}(1+m)^{-\delta}] - r_1 - r_2 - m \tag{28}$$

subject to $r_i \geqslant 0$ (i=1,2) and $m \geqslant 0$.

By solving the first-order conditions of Π_S with respect to r_1, r_2 and m, one has

$$\partial\Pi_S/\partial r_1 = [(\rho_1+\rho_m) u r_1^{-u-1} r_2^{v} - (\rho_2+\rho_m) v r_2^{-u} r_1^{v-1}]\beta(1+m)^{-\delta} - 1 = 0 \tag{29}$$

Table 12
Centralized decision model.

Groups	System profit under decentralized decision				Centralized decision			
	S-Cournot	S-Collusion	N-Cournot	N-Collusion	r_1	r_2	m	Π_S^C
Group 1	1247976	1247202	1248205	1247768	1781	1714	3884	1258738
Group 2	376814	372848	376282	374349	7559	7028	16671	377901
Group 3	117002	116307	116870	116589	712	759	3526	117067
Group 4	899913	901292	858742	692307	126746	118618	408940	927818
Group 5	1384333	1384889	1383734	1382122	2621	2682	15911	1385526
Group 6	1619014	1619093	1618539	1617156	2017	1632	8761	1620287
Group 7	1719270	1719292	1719186	1719159	105	103	328	1719314

$$\partial\Pi_S/\partial r_2 = [-(\rho_1+\rho_m)v r_1^{-u} r_2^{v-1} + (\rho_2+\rho_m)u r_2^{-u-1} r_1^{v}]\beta(1+m)^{-\delta} - 1 = 0 \quad (30)$$

$$\partial\Pi_S/\partial m = [(\rho_1+\rho_m) r_1^{-u} r_2^{v} + (\rho_2+\rho_m) r_2^{-u} r_1^{v}]\beta\delta(1+m)^{-\delta-1} - 1 = 0 \quad (31)$$

From (29) and (30), one can easily derive

$$(r_2/r_1)^{u+v}[u(r_2/r_1)+v] = [(\rho_2+\rho_m)/(\rho_1+\rho_m)][v(r_2/r_1)+u] \quad (32)$$

Let $r_2/r_1 = x$, then (32) can be rewritten as

$$ux^{u+v+1} + vx^{u+v} - [(\rho_2+\rho_m)/(\rho_1+\rho_m)]vx - u[(\rho_2+\rho_m)/(\rho_1+\rho_m)] = 0 \quad (33)$$

Lemma 3. *There exists a unique positive root Θ that satisfies Eq. (33).*

Through the method similar to that presented in the proof of Lemma 1, one can easily derive Lemma 3. Hence, the proof of Lemma 3 is omitted.

Substituting $r_2 = \Theta \cdot r_1$ into (29)–(31) and using simple algebraic operations, one can derive the unique solution to Eqs. (29)–(31), denoted by (r_1^C, r_2^C, m^C), as follows:

$$m^C = [W\beta\delta(T\beta)^{(--u+v)/(u-v+1)}]^{(u-v+1)/(\delta+u-v+1)} - 1 \quad (34)$$

$$r_1^C = (T\beta)^{1/(u-v+1)}(m^C)^{-\delta/(u-v+1)} \quad (35)$$

$$r_2^C = \Theta \cdot r_1^C \quad (36)$$

where $T=(\rho_1+\rho_m)u\Theta^{v}-(\rho_2+\rho_m)v\Theta^{-u}$, $W=(\rho_1+\rho_m)\Theta^{v}+(\rho_2+\rho_m)\Theta^{-u}$, and the superscript "*C*" denotes "**C**entralized decision".

Theorem 9. *The total system profit Π_S reaches its maximum at (r_1^C, r_2^C, m^C).*

Proof. Proof. See Appendix. □

From Theorem 9, we know that $r_i^C (i=1,2)$ and m^C, given by (34)–(36), are the optimal local advertising expenditures and brand name investment under the centralized decision-making system. Substituting (34)–(36) into (28) would yield the maximum total profit of the supply chain system Π_S^C.

Based on the above solution procedure, we compute the optimal centralized advertising policies and corresponding channel profits for the seven groups of parameter values used in the previous section. The results are given in Table 12. One can observe clearly from Table 12 that the integrated advertising policy, (r_1^C, r_2^C, m^C), can improve the total profit of the entire supply chain, as compared to the decentralized advertising policies. Hence, all parties in the supply chain would have incentives to co-operate with each other as long as it is assured that each party in the supply chain can get more profit in cooperation than in the case without cooperation. Thus, to realize the cooperation among all parties in the supply chain, it is necessary to design a suitable way to split the increased system profit incurred by all parties' cooperation. In next section, we will offer a cost-sharing contract to achieve the cooperation of three parties.

6. Supply chain coordination through sharing of local advertising costs

Assume first that the integrated advertising policy (r_1^C, r_2^C, m^C) has been accepted by all participants in the channel. That is, the manufacturer and the duopolistic retailers are all willing to set the brand name investment and local advertising expenditures at m^C and r_i^C, respectively, which is the optimal policy that maximizes the total profit of the centralized channel. Then, for any arbitrarily-given fraction t of local advertising cost shared by the manufacturer, the profits of the manufacturer and the duopolistic retailers would be respectively given by

$$\Pi_M(r_i^C, m^C, t) = \rho_m\left[(\alpha_1+\alpha_2) - \beta(\Theta^{v}+\Theta^{-u})(r_1^C)^{-u+v}(1+m^C)^{-\delta}\right] - t(1+\Theta)r_1^C - m^C \quad (37)$$

$$\Pi_{r1}(r_i^C, m^C, t) = \rho_1\left[\alpha_1 - \beta\Theta^{v}(r_1^C)^{-u+v}(1+m^C)^{-\delta}\right] - (1-t)r_1^C \quad (38)$$

$$\Pi_{r2}(r_i^C, m^C, t) = \rho_2\left[\alpha_2 - \beta\Theta^{-u}(r_1^C)^{-u+v}(1+m^C)^{-\delta}\right] - (1-t)\Theta r_1^C \quad (39)$$

Obviously, supply chain coordination could be implemented, i.e., three parties would accept (r_1^C, r_2^C, m^C), only when they gain profits no less than those in the non-cooperative situation. From (37)–(39), we know that after accepting (r_1^C, r_2^C, m^C) the profits of three partners would depend only on the fraction t of local advertising expenditures shared by the manufacturer. Next, the problem is how to determine a feasible fraction t so that all partners are willing to choose the channel-optimal policy.

For notational convenience, in the latter analysis in this section, let "~" denote "^" or "" and "*k*" denote "*ct*" or "*cn*". Then, the manufacturer is willing to cooperate only if the following inequality is satisfied

$$\Pi_M(r_i^C, m^C, t) \geqslant \Pi_M^{k^*} \quad (40)$$

which gives $t \leqslant t_{max}$, where $t_{max} = \{\rho_m[(\alpha_1+\alpha_2) - \beta(\Theta^{v}+\Theta^{-u})(r_1^C)^{-u+v}(m^C)^{-\delta}] - m^C - \Pi_M^{k^*}\}/[(1+\Theta)r_1^C]$.

And the duopolistic retailers would be interested in adopting the system-optimal policy only if the following condition is satisfied

$$\Pi_{ri}(r_i^C, m^C, t) \geqslant \Pi_{ri}^{k^*} \quad (i=1,2) \quad (41)$$

which gives $t \geqslant t_{min}$, where

$$t_{min} = max\Big\{1 - \Big[\rho_1(\alpha_1 - \beta\Theta^{v}(r_1^C)^{-u+v}(m^C)^{-\delta}) - \Pi_{r1}^{k^*}\Big] \Big/ r_1^C, 1 - \Big[\rho_2(\alpha_2 - \beta\Theta^{-u}(r_1^C)^{-u+v}(m^C)^{-\delta}) - \Pi_{r2}^{k^*}\Big]/r_2^C\Big\}.$$

Therefore, there exists a feasible fraction t only if the following condition holds:

$$0 \leqslant t_{min} \leqslant t_{max} \leqslant 1 \quad (42)$$

If (42) is met, the feasible local-advertising-cost-sharing fraction t accepted by all parties will has to satisfy $t_{min} \leqslant t \leqslant t_{max}$. In other words, to achieve the supply chain coordination, three partners can only negotiate a fraction t from the interval $[t_{min}, t_{max}]$. The value of t trilaterally agreed reflects three parties' negotiation abilities. If $t = t_{max}$, i.e. the duopolistic retailers have absolute dominance over bargaining, all benefits incurred by channel members' coordination would go to the retailers. Contrarily, if $t = t_{min}$, i.e. the manufacturer is dominant in the negotiation, then he would extract most of the increment of channel profit brought by channel members' complete cooperation and leave the rest to the retailer who has more bargaining power than his rival.

Due to the difficulty of describing the negotiation abilities of the supply chain members, it is almost impossible to give a definite value of the cost-sharing fraction t to determine the division of the increased system profit without any further assumptions. Hence, similar to Huang and Li (2001) and Li et al. (2002), we will use the Nash bargaining approach to determine the value of fraction t from the viewpoint of the utility of risk preference.

Let $\Delta\Pi_M = \Pi_M(r_i^C, m^C, t) - \Pi_M^{k^*}$ and $\Delta\Pi_{ri} = \Pi_{ri}(r_i^C, m^C, t) - \Pi_{ri}^{k^*}$ $(i = 1, 2)$. Define $U_M(\Delta\Pi_M)$ and $U_{ri}(\Delta\Pi_{ri})$, which are assumed to be twice differentiable, be the utility functions of the manufacturer and the two retailers, respectively. And define the coefficient of relative risk aversion as follows[5]:

$$R_x(\Delta\Pi_x) = -\Delta\Pi_x \frac{U_x''(\Delta\Pi_x)}{U_x'(\Delta\Pi_x)} \quad (x = M, r_i) \quad (43)$$

where $U_x'(\cdot)$ and $U_x''(\cdot)$ are the first-order and second-order derivative of $U_x(\cdot)$. Instead of adopting the same exponential utility function as that presented in Huang and Li (2001) and Li et al. (2002), we utilize another commonly used function, logarithm function, to measure the utility of each member[6]. By adopting the similar analysis that presented in Huang and Li (2001) and Li et al. (2002), one can easily get each member's optimal share of the increased system profit under the consideration of the risk attitude of the supply-chain members as follows:

$$\Delta\Pi_M^* = (b_m/(b_1 + b_2 + b_m))\Delta\Pi_S \quad (44)$$
$$\Delta\Pi_{r1}^* = (b_1/(b_1 + b_2 + b_m))\Delta\Pi_S \quad (45)$$
$$\Delta\Pi_{r2}^* = (b_2/(b_1 + b_2 + b_m))\Delta\Pi_S \quad (46)$$

where b_m, b_1 and b_2 denote the relative risk aversions of the manufacturer and the two retailers, respectively. (The derivations of (44)–(46) can be seen in Appendix.)

Furthermore, from (37), (40) and (43), one can obtain the following optimal cost-sharing fraction t^*, at which the manufacturer and the two retailers agree to change their optimal advertising policies under decentralized decision-making system to those under centralized decision-making system:

$$t^* = t_{max} - \Delta\Pi_M^*/\big[(1 + \Theta)r_1^C\big] \quad (57)$$

We refer to the above-presented coordination mechanism as ***l***ocal-***a***dvertising-***c***ost-***s***haring contract (abbreviated as *LACS*). From (44)–(47), one can obtain the following implications.

(i) The ratio of each member's increased profit to the other's under *LACS* contract is exactly equal to the ratio of risk-averse degrees for these two members. The higher the extent that one member disgusts risk, the more the share of the increased system profit received by this member. Moreover, if the manufacturer and the two retailers have the same degree of risk aversion, all members in the supply chain will evenly share the increased system profit incurred by channel coordination.

(ii) The optimal *LACS* fraction t^* decreases as the manufacturer's increased profit increases, which implies that the manufacturer with higher degree of risk aversion will share less local advertising costs of the retailers.

Huang and Li (2001) and Li et al. (2002) studied coordination in co-op advertising with one manufacturer and one retailer and showed how to determine a feasible *LACS* fraction t so that both members are willing to choose the centralized decision policy. From the above analysis, one can note that when there are competitive members in the downstream, the coordination mechanism for co-op advertising and the corresponding results presented are distinct from that of Huang and Li (2001) and Li et al. (2002) in the following aspects. For one thing, the range of the feasible *LACS* fraction t will be smaller as compared with the single-manufacturer single-retailer case. For another, when discussing the allocation of increased system profit, if $t = t_{min}$, i.e. the manufacturer is dominant in the negotiation, he can only occupy most of the increment of channel profit brought by channel members' complete cooperation and leave the rest to the retailer who has more bargaining power than his rival, which is different from the works of Huang and Li (2001) and Li et al. (2002) that the manufacturer would take up all the increment of channel profit under such a case.

6.1. Numerical examples

For the sake of convenience, we take Group 4 given in previous section as an example to illustrate the model presented in this section. The parameters of the model are listed below.

$(b_m, b_1, b_2) = (0.2, 0.5, 0.8)$ and $(0.5, 0.5, 0.5)$, other parameters are kept the same as in Group 4.

After using the proposed solution procedure, we can get the feasible negotiating interval of fraction t for *LACS* contract, the optimal *LACS* fraction t^* and the corresponding share of increased system profit under this optimal *LACS* contract for each member. The computed results are shown in Table 13.

[5] The reasons for choosing the risk-averse supply chain members are as follows. First, the empirical studies show that most managers of the firms are risk averse rather than risk neutral or risk seeking (Nadiminti, Mukhopadhyay, & Kriebel, 1996). Second, over the last couple of decades, the business environment has evolved to be increasingly complex that is characterized by high uncertainty and rapid and frequent changes. Any disruption in one firm can rapidly result in a significant adversary impact on the entire chain. At the same time, supply chain members are subject to many potential external sources of disruption, e.g., natural disasters, terrorist attacks, industrial actions, etc., which may lead to increased vulnerability of the chains. What the supply chain managers concern most now is the risk containment or loss minimization for their firms. Thus, the assumption of risk neutral or risk seeking seems to be largely invalid for contemporary supply chain management.

[6] In decision theory, exponential utility implies constant absolute risk aversion with coefficient of absolute risk aversion equal to a constant; while logarithm utility indicates constant relative risk aversion with coefficient of relative risk aversion equal to a constant. The classical theory of risk aversion (see, e.g., Arrow, 1965; Pratt, 1964) has pointed out that the logarithm utility function is considered more plausible than the exponential utility function. For example, in the standard model of one risky asset and one risk-free asset presented in Arrow (1965) and Pratt (1964), the constant absolute risk aversion means that the optimal holding of the risky asset is independent of the level of initial wealth; thus on the margin any additional wealth would be allocated totally to additional holdings of the risk-free asset. This feature explains why the exponential utility function is considered unrealistic.

7. Conclusions

This present paper further extends the existing one-manufacturer–one-retailer co-op advertising models to the situation with a monopolistic manufacturer and two competing retailers. We investigate the impact of four game structures, i.e. Stackelberg–Cournot, Stackelberg–Collusion, Nash–Cournot, and Nash–Collusion, on the local advertising expenditures, brand name investment and local advertising allowance level. Moreover, we develop the centralized-decision model and show that joint decision can improve the performance of the entire supply chain. Finally, we present a *LACS* contract to realize the coordination of the supply chain based on the utility of risk preference. Based on the analysis of the model and results of numerical experiments, we obtain the following insights: (1) In the Stackelberg-• setting, the manufacturer does not always share the two retailers' local advertising costs. The paper identifies the conditions under which the retailers can obtain positive local advertising allowance from the manufacturer in two Stackelberg-• settings. In Huang and Li's (2001) and Li et al.'s (2002) model, however, it is implicitly assumed that the manufacturer always provide positive local advertising allowance in the Stackelberg setting; (2) If the manufacturer offers positive advertising allowance to the retailers in the Stackelberg-• setting, both the manufacturer and the retailers would always prefer the Stackelberg game to the Vertical-Nash game, whether the duopolistic retailers chose Cournot or Collusion. This is contrary to our intuitive expectation; (3) If the manufacturer does not give positive advertising allowance to the retailers in the Stackelberg-• setting, the above preferences of the manufacturer and the two retailers depend on the parameters presented in the model; (4) In the Nash-• setting, no matter which strategy—Cournot or Collusion—the retailers will adopt, the manufacturer would not share any of local advertising costs with the duopolistic retailers; (5) If the Nash game is implemented between two echelons, through collusion the two retailers can force the manufacturer to enhance brand name investment, from which a direct benefit the two retailers obtain is that they can reduce their own local advertising payments.

There are many open research issues that remain to be examined within the framework of cooperative advertising models in a one-manufacturer two-retailer supply chain system. First, while our model focuses on a single manufacturer and two retailers, exactly the same approach can be used to analyze the supply chain coordination with multiple manufacturers and multiple retailers. Second, we assume a deterministic demand faced by each retailer in our model; however, with the rapid improvement of technology, the lifecycles of products become shorter and shorter so that more and more products have the attributes of fashion or seasonal goods (Zhou & Wang, 2009; Wang, Zhou, & Wang, 2010). Thus, a more interesting issue worthy of further research is the value of coordination between both parties when the two retailers selling the seasonal products with random demands.

Table 13
The computed results.

Game structures	$[t_{min}, t_{max}]$	b_m	b_1	b_2	t^*	$\Delta\Pi_M^*$	$\Delta\Pi_{r1}^*$	$\Delta\Pi_{r2}^*$
S-Cournot		0.2	0.5	0.8	0.5215	3721	9302	14883
		0.5	0.5	0.5	0.4988	9302	9302	9302
S-Collusion		0.2	0.5	0.8	0.8406	3537	8842	14147
		0.5	0.5	0.5	0.8190	8842	8842	8842
N-Cournot		0.2	0.5	0.8	0.6156	9210	23025	36841
		0.5	0.5	0.5	0.5593	23025	23025	23025
N-Collusion		0.2	0.5	0.8	0.7252	31401	78504	125606
		0.5	0.5	0.5	0.6333	78504	78504	78504

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Appendix A

A.1. Proof of Theorem 1

Proof. We will prove Theorem 1 in view of the following three cases: $0 \leqslant t_0 < 1$, $t_0 \geqslant 1$ and $t_0 < 0$.

(i) If $\rho_m \geqslant F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$, which implies that $0 \leqslant t_0 < 1$, then $(\hat{t}^{ct^*},\ \hat{m}^{ct^*}) = (t_0, m_0(t_0))$. Let

$$H_{11} = \left.\frac{\partial^2 \Pi_M}{\partial m^2}\right|_{(\hat{t}^{ct^*}, \hat{m}^{ct^*})}, \quad H_{12} = \left.\frac{\partial^2 \Pi_M}{\partial m \partial t}\right|_{(\hat{t}^{ct^*}, \hat{m}^{ct^*})}, \quad H_{22} = \left.\frac{\partial^2 \Pi_M}{\partial t^2}\right|_{(\hat{t}^{ct^*}, \hat{m}^{ct^*})},$$

one can easily get

$$H_{11} = -[(\delta + u - v + 1)/(tu - v + 1)](1 + \hat{m}^{ct^*})^{-1}, H_{12} = 0,$$
$$H_{22} = -[(1 + \Phi)/(u - v + 1)](H\beta)^{1/(u-v+1)}(1 - \hat{t}^{ct^*})^{-(2u-2v+3)/(u-v+1)}.$$

It can be observed that $H_{11} < 0$, $H_{22} < 0$ and $H_{11}H_{22} - (H_{12})^2 > 0$. Hence, the Hessian matrix of the manufacturer's profit, Π_M, is negative definite at $(\hat{t}^{ct^*}, \hat{m}^{ct^*})$. Due to the fact that the first-order conditions of Π_M with respect to t and m reveal the unique positive root $(\hat{t}^{ct^*}, \hat{m}^{ct^*})$, so $(\hat{t}^{ct^*}, \hat{m}^{ct^*})$ is the maximum point of Π_M.

(ii) If $\rho_m \leqslant F_1(\rho_1, \rho_2)$, which means that $t_0 \geqslant 1$, then it is certain for the manufacturer to set his shared fraction of local advertising expenditure level as $\hat{t}^{ct^*} = 1$ to maximize his profit. From (13), we know that the equilibrium brand name investment of the manufacturer in the Stackelberg–Cournot setting should be given as

$$\hat{m}^{ct^*} = [(H\beta)^{1/(u-v+1)}(1 + \Phi)(1 - \hat{t}^{ct^*})^{-1/(u-v+1)}\delta / (u - v)]^{(u-v+1)/(\delta+u-v+1)} - 1 \qquad \text{(A1)}$$

However, from (A1) and (11), one can derive that

$$\lim_{\hat{t}^{ct^*} \to 1} \hat{m}^{ct^*} = +\infty \text{ and } \lim_{\hat{t}^{ct^*} \to 1} \Pi_M = -\infty.$$

Thus, the manufacturer will obtain negative profit at his equilibrium allowance policy $\hat{t}^{ct^*} = 1$. This case is possible to happen in the sense that all the parties in the supply chain share symmetric information. Therefore, the retailers are aware of the manufacturer's cost structure and his optimal strategies. As a result, if $t_0 \geqslant 1$, the two retailers will be sure to increase their local advertising investments infinitely and leave all the local advertising costs to the manufacturer. Hence, as a rational person, the manufacturer will not do business with the retailers in this case.

(iii) If $F_1(\rho_1,\ \rho_2) < \rho_m < F_1(\rho_1, \rho_2)[1 + 1/(u - v)]$, which indicates that $t_0 < 0$, as shown in Huang et al. (2001), the equilibrium local advertising allowance of the manufacturer must equal zero, i.e. $\hat{t}^{ct^*} = 0$. Substituting $\hat{t}^{ct^*} = 0$ into (A1) gives the corresponding optimal brand name investment $\hat{m}^{ct^*} = m_0(0)$. Hence, $(0, m_0(0))$ is the equilibrium policies of the manufacturer, at which Π_M reaches its maximum. □

A.2. Proof of Lemma 1

Proof. Let $f(x) = ux^{u+v+1}+vx^{u+v} -(\rho_2/\rho_1)vx-u(\rho_2/\rho_1)$, taking the first- and second-order derivatives of $f(x)$ with respect to x gives respectively

$$f'(x) = u(u+v+1)x^{u+v} + v(u+v)x^{u+v-1} - (\rho_2/\rho_1)v \quad (A2)$$

$$f''(x) = u(u+v)(u+v+1)x^{u+v-1} + v(u+v)(u+v-1)x^{u+v-2} \quad (A3)$$

Since $0 < v < u$ and $u+v > 1$, from (A3) we have $f''(x) > 0$. Additionally, from (A2) one can easily derive

$f'(x=0) = -(\rho_2/\rho_1)v < 0$ and $\lim_{x\to+\infty} f'(x) = +\infty$.

Since $f''(x) > 0$, i.e. $f'(x)$ is a monotone increasing function of x, thus there exists a unique positive root x_0 to the equation $f'(x) = 0$, at which $f(x)$ reaches its minimum.

Notice that $f'(x_0) = u(u+v+1)x_0^{u+v} + v(u+v)x_0^{u+v-1} - (\rho_2/\rho_1)v = 0$. then

$$(\rho_2/\rho_1)v = u(u+v+1)x_0^{u+v} + v(u+v)x_0^{u+v-1} \quad (A4)$$

and the minimum of

$$f(x) \text{ is } f(x_0) = ux_0^{u+v+1} + vx_0^{u+v} - (\rho_2/\rho_1)vx_0 - u(\rho_2/\rho_1) \quad (A5)$$

substituting (A4) into (A5), one can derive

$$f(x_0) = x_0[-u(u+v)x_0^{u+v} - v(u+v-1)x_0^{u+v-1}] - u(\rho_2/\rho_1) \quad (A6)$$

Because $0 < v < u$ and $u+v > 1$, we have $f(x_0) < 0$. In addition, it is easy to check that $f(x=0) = -u\,(\rho_2/\rho_1) < 0$ and $\lim_{x\to+\infty} f(x) = +\infty$. Hence, there exists a unique positive root, denoted by Ψ, to the equation $f(x) = 0$. □

A.3. Proof of Lemma 2

Proof. Let $g(x)=(vx+u)/(ux+v)$, by taking the first-order derivative of $g(x)$ with respect to x, one has

$g'(x) = (v^2 - u^2)/(ux+v)^2$. Since $u > v$, $g'(x) < 0$, i.e. $g(x)$ is a monotone decreasing function of x. From Lemma 1, we know that $\Psi > 0$. Hence, one can obtain $g(x = \Psi) < g(x = 0)$, i.e. $(v\Psi + u)/(u\Psi + v) < u/v$. The proof of Lemma 2 is complete. □

A.4. Proof of Theorem 2

Proof. Let $H_{11} = \frac{\partial^2 \Pi_r}{\partial r_1^2}|_{(\hat{r}_1^{cn},\hat{r}_2^{cn})}, H_{12} = \frac{\partial^2 \Pi_r}{\partial r_1 \partial r_2}|_{(\hat{r}_1^{cn},\hat{r}_2^{cn})}, H_{22} = \frac{\partial^2 \Pi_r}{\partial r_2^2}|_{(\hat{r}_1^{cn},\hat{r}_2^{cn})}$, from (15) and (16), one can derive that

$$H_{11} = -\left[\rho_1 u(u+1)(\hat{r}_1^{cn})^{-u-2}(\hat{r}_2^{cn})^{v} + \rho_2 v(v-1)(\hat{r}_2^{cn})^{-u}(\hat{r}_1^{cn})^{v-2}\right] \beta(1+m)^{-\delta} \quad (A7)$$

$$H_{22} = -\left[\rho_1 v(v-1)(\hat{r}_1^{cn})^{-u}(\hat{r}_2^{cn})^{v-2} + \rho_2 u(u+1)(\hat{r}_2^{cn})^{-u-2}(\hat{r}_1^{cn})^{v}\right] \beta(1+m)^{-\delta} \quad (A8)$$

$$H_{12} = -\left[\rho_1 (\hat{r}_1^{cn})^{-u-1}(\hat{r}_2^{cn})^{v-1} + \rho_2 (\hat{r}_2^{cn})^{-u-1}(\hat{r}_1^{cn})^{v-1}\right] uv\beta(1+m)^{-\delta} \quad (A9)$$

From (15), we have

$$\rho_1 u(\hat{r}_1^{cn})^{-u-1}(\hat{r}_2^{cn})^{v} - \rho_2 v(\hat{r}_2^{cn})^{-u}(\hat{r}_1^{cn})^{v-1} = (1-t)/[\beta(1+m)^{-\delta}] > 0 \quad (A10)$$

Substituting (A10) into (A7), we get

$$H_{11} < -\left[\rho_2 (u+1) v(\hat{r}_1^{cn})^{v-2}(\hat{r}_2^{cn})^{-u} + \rho_2 v(v-1)(\hat{r}_2^{cn})^{-u}(\hat{r}_1^{cn})^{v-2}\right] \beta(1+m)^{-\delta} = -\rho_2 v(u+v)(\hat{r}_1^{cn})^{v-2}(\hat{r}_2^{cn})^{-u} < 0 \quad (A11)$$

From (A7)–(A9), one can easily derive

$$H_{11}H_{22} - (H_{12})^2 = \beta^2(1+m)^{-2\delta}(u-v+1)(\hat{r}_1^{cn})^{-2u+2v-4}\Psi^{-u+v-2} \times \{\rho_1\rho_2[(u^2-v^2)(u+v)+u^2+v^2] - uv[\rho_1^2\Psi^{u+v}+\rho_2^2\Psi^{-u-v}]\} \quad (A12)$$

Because of (19) and (A10), we know

$$\Psi^{-u-v} = (\hat{r}_2^{cn}/\hat{r}_1^{cn})^{-u-v} < \rho_1 u/(\rho_2 v) \quad (A13)$$

From (17), there is

$$\Psi^{u+v} = \rho_2(v\Psi + u)/[\rho_1(u\Psi + v)] \quad (A14)$$

By Lemma 2 and (A14), one can easily obtain

$$\Psi^{u+v} < \rho_2 u/(\rho_1 v) \quad (A15)$$

Substituting (A13) and (A15) into (A12), one get

$$H_{11}H_{22} - (H_{12})^2 > \beta^2(1+m)^{-2\delta}(u-v+1)(\hat{r}_1^{cn})^{-2u+2v-4}(\Psi)^{-u+v-2} \cdot \{\rho_1\rho_2[(u^2-v^2)(u+v)+u^2+v^2] - uv[\rho_1^2\rho_2 u/(\rho_1 v) + \rho_2^2\rho_1 u/(\rho_2 v)]\} = \beta^2(1+m)^{-2\delta}(u-v+1)(\hat{r}_1^{cn})^{-2u+2v-4}\Psi^{-u+v-2}\rho_1\rho_2 \times (u^2-v^2)(u+v-1)$$

Since $0 < v < u$ and $u + v > 1$, there is

$$H_{11}H_{22} - (H_{12})^2 > 0 \quad (A16)$$

Hence, from (A11) and (A16), we know that the Hessian matrix of Π_r is negative definite at $(\hat{r}_1^{cn}, \hat{r}_2^{cn})$. Due to the uniqueness of $(\hat{r}_1^{cn}, \hat{r}_2^{cn})$ for (15) and (16), then Π_r reaches its maximum at $(\hat{r}_1^{cn}, \hat{r}_2^{cn})$. The proof of Theorem 2 is complete. □

A.5. Derivation of equilibrium solution in the Nash–Collusion setting

First, it is obvious that the manufacturer's optimal shared fraction of local advertising expense, t, should be zero because of its negative coefficient in the objective. Since $\partial^2\Pi_M/\partial m^2 < 0$, $\partial^2\Pi_{r1}/\partial r_1^2 < 0$ and $\partial^2\Pi_{r2}/\partial r_2^2 < 0$, then solving the first-order conditions easily gives the equilibrium advertising scheme as

$$\bar{r}_1^{cn^*} = [\rho_m\beta\delta(\Psi^v + \Psi^{-u})]^{-\delta/(\delta+u-v+1)}(G\beta)^{(\delta+1)/(\delta+u-v+1)}, \quad \bar{r}_2^{cn^*} = \Psi\cdot\bar{r}_1^{cn^*},$$
$$\bar{t}^{cn^*} = 0, \quad \bar{m}^{cn^*} = [\rho_m\beta\delta(\Psi^v + \Psi^{-u})]^{(u-v+1)/(\delta+u-v+1)} \times (G\beta)^{(-u+v)/(\delta+u-v+1)} - 1$$

A.6. Proof of Theorem 4

Proof.

(1) If $\rho_m \geqslant \rho u\,[1 + 1/(u - v)]$, then Theorem 4(1)-(i) can be easily proved by simple comparison of the results in Table 1. Hence, the proof of Theorem 4(1)-(i) is omitted for simplicity.
(ii) We first prove $\widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*}$. Known from the 5th row of Table 2, in order to prove $\widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*}$, we need to prove

$$-(\rho_m - \rho u)^{-(\delta+u-v)/(\delta+u-v+1)}(1+u) < [\rho_m - \rho(u-v)]^{-(\delta+u-v)/(\delta+u-v+1)}[1+u-v] \quad (A17)$$

Let $f_1(x) = -(\rho_m - \rho x)^{-(\delta + u-v)/(\delta + u-v+1)}(1 + x)$, it is easy to check that $f_1'(x) < 0$, which means that $f_1(x)$ is a monotone decreasing function over the whole domain of x. As a result, it is decreasing in the interval $[u - v, u]$. Hence, one has $f_1(u) < f_1(u - v)$, which implies that (A17) holds, i.e. the inequality $\widehat{\Pi}_{ri}^{ct^*} < \widehat{\Pi}_{ri}^{cn^*}$ is satisfied.

Next, we prove $\widehat{\Pi}_M^{ct^*} > \widehat{\Pi}_M^{cn^*}$.

The proof of $\widehat{\Pi}_M^{ct^*} > \widehat{\Pi}_M^{cn^*}$ can be obtained by comparing the corresponding results presented in the 6th row of Table 2.

Finally, we prove $\widehat{\Pi}_S^{ct^*} < \widehat{\Pi}_S^{cn^*}$.

Similarly, known from the 7th row of Table 2, to prove $\widehat{\Pi}_S^{ct^*} < \widehat{\Pi}_S^{cn^*}$ is equivalent to proving

$$-[\rho(1+u)+(\rho_m-\rho u)(\delta+u-v+1)] \times(\rho_m-\rho u)^{-(\delta+u-v)/(\delta+u-v+1)} < -[\rho(1+u-v) + (\rho_m-\rho(u-v))(\delta+u-v+1)] \times[\rho_m-\rho(u-v)]^{-(\delta+u-v)/(\delta+u-v+1)} \quad (A18)$$

Let $f_2(x) = -[\rho(1 + x)+(\rho_m - \rho x)(\delta + u - v + 1)](\rho_m - \rho x)^{-(\delta + u-v)/(\delta + u-v + 1)}$. Taking the first-order derivative of $f_2(x)$ with respect to x, and one has

$$f_2'(x) = -(\rho_m-\rho x)^{-[2(\delta+u-v)+1]/(\delta+u-v+1)}\rho^2(1+x)(\delta+u - v)/(\delta+u-v+1)$$

Due to $\rho_m \geqslant \rho u\ [1 + 1/(u - v)]$ and $u > v > 0$, one obtains that $f_2'(x) < 0$ when $x \in [u - v, u]$. Hence, we have $f_2(u) < f_2(u - v)$, which indicates that (A18) is satisfied. The proof of Theorem 4(1) is thus complete.

(2) If $\rho u < \rho_m < \rho_m u/(u - v) < \rho u\ [1 + 1/(u - v)]$, then Theorem 4(2) can be easily proved by simple comparison of the results in Table 3. Hence, the proof of Theorem 4(2) is omitted for simplicity.

(3) If $\rho u < \rho_m < \rho u[1 + 1/(u - v)]$ and $\rho_m \geqslant \rho(u - v)[1 + 1/(u - v)]$, then Theorem 4(3) can be easily proved by comparing the corresponding results presented in Table 4. Hence, the proof of Theorem 4(3) is omitted for simplicity. The proof of Theorem 4 is thus complete. □

A.7. Proof of Theorem 5

Proof.

(i) Known from rows 2–4 of Table 5, the results of Theorem 5(i) can be easily obtained by comparison.

(ii) We first prove $\overline{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{cn^*}$. Known from the 5th row of Table 5, the proof of $\overline{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{cn^*}$ is equivalent to the proof of

$$-u^{(-u+v)/(\delta+u-v+1)}(1+u) < -(u-v)^{(-u+v)/(\delta+u-v+1)}(1+u-v) \quad (A19)$$

Let $g_1(x) = -x^{(-u+v)/(\delta+u-v+1)}(1 + x)$. Taking the first-order derivative of $g_1(x)$ with respect to x, one has

$$g_1'(x) = -x^{(-u+v)/(\delta+u-v+1)-1}[x(\delta+1)-(u-v)]/(\delta+u-v+1) \quad (A20)$$

From (A20), one can easily get $g_1'(x) < 0$ for $u - v \leqslant x \leqslant u$, which indicates that $g_1(x)$ is a monotone decreasing function of x in $[u - v, u]$. Therefore, one has $g_1(u) < g_1(u - v)$, i.e. (A19) is valid. Thus, we have $\overline{\Pi}_{ri}^{ct^*} < \overline{\Pi}_{ri}^{cn^*}$.

Next, we prove $\overline{\Pi}_M^{ct^*} > \overline{\Pi}_M^{cn^*}$.

The proof of $\overline{\Pi}_M^{ct^*} > \overline{\Pi}_M^{cn^*}$ can be easily accomplished by simply comparing the results presented in Table 5.

Finally, we prove $\overline{\Pi}_S^{ct^*} > \overline{\Pi}_S^{cn^*}$.

Known from the 7th line of Table 5, to prove $\overline{\Pi}_S^{ct^*} > \overline{\Pi}_S^{cn^*}$ is equivalent to proving

$$-u^{(-u+v)/(\delta+u-v+1)}[\rho(1+u)+\rho_m(1+\delta)] > -(u-v)^{(-u+v)/(\delta+u-v+1)}[\rho(1+u-v)+\rho_m(1+\delta)] \quad (A21)$$

Let $g_2(x) = -x^{(-u+v)/(\delta+u-v+1)}[\rho(1 + x) + \rho_m(1 + \delta)]$. Taking the first-order derivative of $g_2(x)$ with respect to x, one has

$$g_2'(x) = -x^{(-u+v)/(\delta+u-v+1)-1}\{\rho x(1+\delta)-(u-v)[\rho + \rho_m(1+\delta)]\}/(\delta+u-v+1) \quad (A22)$$

Due to $\rho u(1 + \delta) - (u - v)[\rho + \rho_m(1 + \delta)] = [\rho u - \rho_m(u - v)](1 + \delta) - \rho(u - v) < 0$, then one can easily verify that $\rho x(1 + \delta) - (u - v)[\rho + \rho_m(1 + \delta)] < 0$ for $u - v \leqslant x \leqslant u$. Thus, from (A22), one can obtain $g_2'(x) > 0$ with $u - v \leqslant x \leqslant u$, which means that $g_2(x)$ is a monotone increasing function of x in $[u - v, u]$. Hence, one has $g_2(u) > g_2(u - v)$, which is equivalent to (A21).

The proof of Theorem 5 is completed. □

A.8. Proof of Theorem 6

Proof.

(1) If $\rho_m \geqslant \rho u\ [1 + 1/(u - v)]$, then

(i) First, it is easy to verify that $\hat{t}^{ct^*} \geqslant \bar{t}^{ct^*}$ from the 2nd rows of Tables 2 and 5. Next, we prove $\hat{m}^{ct^*} < \bar{m}^{ct^*}$. Known from the 3rd rows of Tables 2 and 5, the proof of $\hat{m}^{ct^*} < \bar{m}^{ct^*}$ is equivalent to proving

$$[(\rho_m-\rho u)(u-v)]^{1/(u-v+1)}/(u-v) < \rho_m(\rho u)^{(-u+v)/(u-v+1)} \quad (A23)$$

As a result of $\rho_m \geqslant \rho u[1 + 1/(u - v)]$, one has

$$(\rho_m-\rho u)(u-v)/(\rho u) > 1 \quad (A24)$$

From (A24), one can derive

$$\begin{aligned}&[(\rho_m-\rho u)(u-v)]^{1/(u-v+1)}/(u-v)\\ &= [\rho u(\rho_m-\rho u)(u-v)/\rho u]^{1/(u-v+1)}/(u-v)\\ &< [(\rho_m-\rho u)(u-v)/(\rho u)](\rho u)^{1/(u-v+1)}/(u-v)\\ &= (\rho_m-\rho u)(\rho u)^{(-u+v)/(u-v+1)} < \rho_m(\rho u)^{(-u+v)/(u-v+1)},\end{aligned}$$

which verifies (A23). Thus, the proof of $\hat{m}^{ct^*} < \bar{m}^{ct^*}$ is complete.

Finally, we prove $\hat{r}_i^{ct^*} > \bar{r}_i^{ct^*}$.

Let Δ_1 denote the ratio of $\hat{r}_i^{ct^*}$ with respect to $\bar{r}_i^{ct^*}$. From the 4th rows of Tables 2 and 5, one has

$$\begin{aligned}\Delta_1 &= \{(u-v)^{(\delta+1)}(\rho_m-\rho u)/[(\rho_m)^{-\delta}(\rho u)^{\delta+1}]\}^{1/(\delta+u-v+1)}\\ &= \{[(\rho_m-\rho u)/\rho_m][\rho u/(\rho_m(u-v))]^{-(\delta+1)}\}^{1/(\delta+u-v+1)}\end{aligned} \quad (A25)$$

From (A24), we have

$$0 < \rho u/[\rho_m(u-v)] < 1 \text{ and } \rho_m - \rho u > \rho u/(u-v) \quad (A26)$$

Substituting (A26) into (A25) gives

$$\begin{aligned}\Delta_1 &> \{[\rho u/(\rho_m(u-v))][\rho u/(\rho_m(u-v))]^{-(\delta+1)}\}^{1/(\delta+u-v+1)}\\ &= \{\rho u/[\rho_m(u-v)]\}^{-\delta/(\delta+u-v+1)} > 1,\end{aligned}$$

which indicates that $\hat{r}_i^{ct^*} > \bar{r}_i^{ct^*}$.

The proof of Theorem 6(1)(i) is complete.

(ii) We first prove $\widehat{\Pi}_{ri}^{ct^*} > \overline{\Pi}_{ri}^{ct^*}$.

Let Δ_2 denote the ratio of $\widehat{\Pi}_{ri}^{ct^*}$ with respect to $\overline{\Pi}_{ri}^{ct^*}$. From the 5th rows of Tables 2 and 5, we have

$$\begin{aligned}\Delta_2 &= \left\{(u-v)^{(u-v)}(\rho_m-\rho u)^{\delta+u-v}/\left[\rho_m^{\delta}(\rho u)^{u-v}\right]\right\}^{-1/(\delta+u-v+1)}\\ &= \{[(\rho_m-\rho u)/\rho_m]^{\delta}[(\rho_m-\rho u)(u-v)/(\rho u)]^{-(u-v)}\}^{-1/(\delta+u-v+1)}\end{aligned}$$

Because of (A24) and $(\rho_m\text{-}\rho u)/\rho_m < 1$, one can easily derive $\Delta_2 > 1$, which implies that $\widehat{\Pi}_{ri}^{ct^*} > \overline{\Pi}_{ri}^{ct^*}$.
Next, we prove $\widehat{\Pi}_{M}^{ct^*} > \overline{\Pi}_{M}^{ct^*}$.
It is easy to verify that $\widehat{\Pi}_{M}^{ct^*} > \overline{\Pi}_{M}^{ct^*}$ by using a similar method as that presented in the proof of $\widehat{\Pi}_{ri}^{ct^*} > \overline{\Pi}_{ri}^{ct^*}$. Hence, this proof is omitted.
Finally, we prove $\widehat{\Pi}_{S}^{ct^*} > \overline{\Pi}_{S}^{ct^*}$.
Due to the fact that $\widehat{\Pi}_{ri}^{ct^*} > \overline{\Pi}_{ri}^{ct^*}$ and $\widehat{\Pi}_{M}^{ct^*} > \overline{\Pi}_{M}^{ct^*}$, we get

$$\widehat{\Pi}_{S}^{ct^*} = \widehat{\Pi}_{r1}^{ct^*} + \widehat{\Pi}_{r2}^{ct^*} + \widehat{\Pi}_{M}^{ct^*} > \overline{\Pi}_{r1}^{ct^*} + \overline{\Pi}_{r2}^{ct^*} + \overline{\Pi}_{M}^{ct^*} = \overline{\Pi}_{S}^{ct^*}.$$

The proof of Theorem 6(1) is thus complete.

(2) If $\rho u < \rho_m < \rho_m u/(u-v) < \rho u\ [1 + 1/(u-v)]$ or $\rho u < \rho_m < \rho u[1 + 1/(u-v)]$ and $\rho_m \geqslant \rho(u\text{-}v)[1 + 1/(u-v)]$, then Theorem 6(2) can be easily proved by comparing the corresponding results presented in Table 3 (or Table 4) and Table 5. Hence, the proof of Theorem 6(2) is omitted for simplicity.

The proof of Theorem 6 is thus complete. □

A.9. Proof of Theorem 7

Proof. Theorem 7 can be proved by adopting the same method as that used in the proof of Theorem 6. Hence, the proof of Theorem 7 is omitted for simplicity. □

A.10. Proof of Theorem 8

Proof. We will prove Theorem 8 according to the following three steps.

(i) We will prove that if $\rho_2/\rho_1 \geqslant 1$, then $\Phi \leqslant \Psi$; otherwise, then $\Phi > \Psi$. From (10) and (21), Φ and Ψ can be given respectively by

$$\Phi = (\rho_2/\rho_1)^{1/(u+v+1)} \tag{A27}$$

$$u\Psi^{u+v+1} + v\Psi^{u+v} - (\rho_2/\rho_1)v\Psi - u(\rho_2/\rho_1) = 0 \tag{A28}$$

Let $f(x) = ux^{u+v+1} + vx^{u+v} - (\rho_2/\rho_1)vx - u(\rho_2/\rho_1)$, from the proof of Lemma 1, one can obtain that if $0 < x < \Psi$, then $f(x) < 0$; if $x = \Psi$, then $f(x) = 0$; if $x > \Psi$, then $f(x) > 0$. However, from (A28), one has

$$f(x=\Phi) = v(\rho_2/\rho_1)^{(u+v)/(u+v+1)}[1 - (\rho_2/\rho_1)^{2/(u+v+1)}] \tag{A29}$$

Obviously, if $\rho_2/\rho_1 \geqslant 1$, then $f(x = \Phi) \leqslant 0$, which means $\Phi \leqslant \Psi$; if $\rho_2/\rho_1 < 1$, then $f(x = \Phi) > 0$, which implies $\Phi > \Psi$.
The proof of (i) is complete.

(ii) We will prove that $t_0 < t_1$. Because of (12) and (23), the proof of $t_0 < t_1$ is equivalent to the proof of the following inequality:

$$\begin{aligned}&(1+\Phi)/[(\Phi^{v}+\Phi^{-u})\rho_m/(\rho_1 u\Phi^{v}) - (1+\Phi)]\\ &\quad > (1+\Psi)/[(\Psi^{v}+\Psi^{-u})\rho_m/(\rho_1 u\Psi^{v} - \rho_2 v\Psi^{-u})\\ &\qquad - (1+\Psi)]\end{aligned} \tag{A30}$$

First, we will show that (A30) is satisfied with $\rho_2/\rho_1 < 1$.
Further algebraic operations for the right side of (A30) gives

$$\begin{aligned}&(1+\Psi)/[(\Psi^{v}+\Psi^{-u})\rho_m/(\rho_1 u\Psi^{v} - \rho_2 v\Psi^{-u}) - (1\\ &\quad + \Psi)]\\ &\quad < (1+\Psi)/[(\Psi^{v}+\Psi^{-u})\rho_m/(\rho_1 u\Psi^{v}) - (1+\Psi)]\\ &\quad = (1+\Psi)/[(1+\Psi^{-u-v})\rho_m/(\rho_1 u) - (1+\Psi)]\end{aligned} \tag{A31}$$

Let $h_1(x) = (1 + x)/[(1 + x^{-u-v})\rho_m/(\rho_1 u) - (1 + x)]$. Taking the first-order derivative of $h_1(x)$ with respect to x yields

$$\begin{aligned}h_1'(x) &= [\rho_m/(\rho_1 u)][(1+x^{-u-v}) + (u+v)(1+x)x^{-u-v-1}]\\ &\quad /[\rho_m(1+x^{-u-v})/(\rho_1 u) - (1+x)]^2\end{aligned} \tag{A32}$$

It is obvious that $h_1'(x) > 0$ for $x > 0$. Since $\rho_2/\rho_1 < 1$, from (i) mentioned above, we know $\Phi > \Psi$. Hence, one can derive

$$h_1(\Psi) < h_1(\Phi) \tag{A33}$$

From (A31) and (A33), one can easily verify that (A30) with $\rho_2/\rho_1 < 1$.
Next, we will point out that (A30) still applies to the case with $\rho_2/\rho_1 \geqslant 1$.
Since $\hat{r}_2^{cn} = \Psi \cdot \hat{r}_1^{cn}$, from (15) and (16), one easily obtains

$$\rho_1 u\Psi^{v} - \rho_2 v\Psi^{-u} = \rho_2 u\Psi^{-u-1} - \rho_1 v\Psi^{v-1} \tag{A34}$$

Hence, the right hand side of (A30) can be simplified to

$$\begin{aligned}&(1+\Psi)/[(\Psi^{v}+\Psi^{-u})\rho_m/(\rho_1 u\Psi^{v} - \rho_2 v\Psi^{-u}) - (1+\Psi)]\\ &\quad = (1+\Psi)/[(\Psi^{v}+\Psi^{-u})\rho_m/(\rho_2 u\Psi^{-u-1} - \rho_1 v\Psi^{v-1})\\ &\qquad - (1+\Psi)] < (1+\Psi)/\{(\Psi^{v}+\Psi^{-u})\rho_m/[\rho_2 u\Psi^{-u-1}]\\ &\qquad - (1+\Psi)\} = 1/[(\rho_m/\rho_2 u)(\Psi^{u+v+1}+\Psi)/(1+\Psi) - 1]\end{aligned} \tag{A35}$$

From (A27), one has $\rho_1/\rho_2 = \Phi^{-(u+v+1)}$. Because of

$$\begin{aligned}\rho_m/(\rho_1 u\Phi^{v}) &= (\rho_m/\rho_2 u)/[(\rho_1/\rho_2)\Phi^{v}]\\ &= \rho_m/(\rho_2 u\Phi^{-u-1})\end{aligned} \tag{A36}$$

the left hand side of (A30) can be rewritten as

$$\begin{aligned}&(1+\Phi)/[(\Phi^{v}+\Phi^{-u})\rho_m/(\rho_1 u\Phi^{v}) - (1+\Phi)]\\ &\quad = (1+\Phi)/[(\Phi^{v}+\Phi^{-u})\rho_m/(\rho_2 u\Phi^{-u-1}) - (1+\Phi)]\\ &\quad = 1/[(\rho_m/\rho_2 u)(\Phi^{u+v+1}+\Phi)/(1+\Phi) - 1]\end{aligned} \tag{A37}$$

Let $h_2(x) = (x^{u+v+1}+x)/(1 + x)$, it can be easily shown that $h_2'(x) > 0$ for $x > 0$. Since $\rho_2/\rho_1 \geqslant 1$, from (i) above, one has $\Phi \leqslant \Psi$. Hence, one can derive

$$h_2(\Phi) < h_2(\Psi) \tag{A38}$$

From (A35), (A37) and (A38), one can easily verify that (A30) still holds with $\rho_2/\rho_1 \geqslant 1$. The proof of (ii) is complete.

(iii) $\hat{t}^{ct^*} \leqslant \hat{t}^{cn^*}$From Theorems 1 and 3, we know that $\hat{t}^{ct^*}$ and $\hat{t}^{cn^*}$ are determined by t_0 and t_1, respectively. From Theorem 1, we know that if $(\Phi^{v}+\Phi^{-u})\rho_m/H > (1+ \Phi)[1 + 1/(u-v)]$, then $1 > t_0 > 0$. Due to (ii) mentioned above, we further have $t_1 > t_0$. With the proof of Theorems 1 and 3, one can easily obtain that $\hat{t}^{ct^*} < \hat{t}^{cn^*}$. If $(1 + \Phi) < (\Phi^{v} + \Phi^{-u})\rho_m/H < (1 + \Phi)[1 + 1/(u-v)]$, then $t_0 = 0$, whereas the value of t_1 may be equal to or greater than zero under such condition, which means that $\hat{t}^{ct^*} \leqslant \hat{t}^{cn^*}$.

The proof of Theorem 8 is complete. □

A.11. Proof of Theorem 9

Proof. By taking the second-order partial derivative of Π_S with respect to m, one can obtain

$$\begin{aligned}\partial^2\Pi_S/\partial m^2 &= -[(\rho_1+\rho_m)r_1^{-u}r_2^{v} + (\rho_2+\rho_m)r_2^{-u}r_1^{v}]\beta\delta(\delta+1)\\ &\quad \times (1+m)^{-\delta-2}\end{aligned} \tag{A39}$$

Obviously, from (A39), one has $\partial^2\Pi_S/\partial m^2 < 0$. Hence, for any given r_1 and r_2, the optimal brand name investment can be obtained by solving Eq. (30), which gives

$$m^C = \{[(\rho_1+\rho_m)r_1^{-u}r_2^{v} + (\rho_2+\rho_m)r_2^{-u}r_1^{v}]\beta\delta\}^{1/(\delta+1)} - 1 \tag{A40}$$

Substituting (A40) into (27), the system's profit maximization problem can be formulated as

$$\max_{r_1,r_2} \Pi_S = (\rho_1+\rho_m)\alpha_1 + (\rho_2+\rho_m)\alpha_2 - \left[(\rho_1+\rho_m)r_1^{-u}r_2^{v} + (\rho_2+\rho_m)r_2^{-u}r_1^{v}\right]^{1/(\delta+1)}A - r_1 - r_2 \quad (A41)$$

where $A = (\beta\delta)^{1/(\delta+1)}(1+1/\delta)$.

In the following, we will show that Π_S reaches its maximum at (r_1^C, r_2^C).

Let $H_{11} = \frac{\partial^2\Pi_S}{\partial r_1^2}|_{(r_1^C,r_2^C)}, H_{12} = \frac{\partial^2\Pi_S}{\partial r_1\partial r_2}|_{(r_1^C,r_2^C)}, H_{22} = \frac{\partial^2\Pi_S}{\partial r_2^2}|_{(r_1^C,r_2^C)}$,

one can easily derive that

$$H_{11} = [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)][C^2\delta/(\delta+1) - BD] \quad (A42)$$

$$H_{22} = [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)][E^2\delta/(\delta+1) - BF] \quad (A43)$$

$$H_{12} = [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)][CE\delta/(\delta+1) + BL] \quad (A44)$$

where

$$B = (\rho_1+\rho_m)(r_1^C)^{-u}(r_2^C)^{v} + (\rho_2+\rho_m)(r_2^C)^{-u}(r_1^C)^{v},$$

$$C = (\rho_1+\rho_m)u(r_1^C)^{-u-1}(r_2^C)^{v} - (\rho_2+\rho_m)v(r_2^C)^{-u}(r_1^C)^{v-1},$$

$$D = (\rho_1+\rho_m)u(u+1)(r_1^C)^{-u-2}(r_2^C)^{v} + (\rho_2+\rho_m)v(v-1)(r_2^C)^{-u}(r_1^C)^{v-2},$$

$$E = -(\rho_2+\rho_m)u(r_2^C)^{-u-1}(r_1^C)^{v} + (\rho_1+\rho_m)v(r_1^C)^{-u}(r_2^C)^{v-1},$$

$$F = (\rho_2+\rho_m)u(u+1)(r_2^C)^{-u-2}(r_1^C)^{v} + (\rho_1+\rho_m)v(v-1)(r_1^C)^{-u}(r_2^C)^{v-2},$$

$$L = \left[(\rho_1+\rho_m)(r_1^C)^{-u-1}(r_2^C)^{v-1} + (\rho_2+\rho_m)(r_2^C)^{-u-1}(r_1^C)^{v-1}\right]uv$$

From (28), one obtains

$$\Theta^{u+v} = (r_2^C/r_1^C)^{u+v} > (\rho_2+\rho_m)v/[(\rho_1+\rho_m)u] \quad (A45)$$

Substituting (44) and (A45) into (A42), one has

$$\begin{aligned} H_{11} &\leqslant [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)][C^2 - BD] \\ &= [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)](r_1^C)^{-2u+2v-2}\Theta^{-u+v}\cdot\{-(\rho_1+\rho_m)^2u\Theta^{u+v} + (\rho_2+\rho_m)^2v\Theta^{-u+v} - (\rho_1+\rho_m)(\rho_2+\rho_m)\times[2uv+u(u+1)+v(v-1)]\} \\ &\leqslant [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)](r_1^C)^{-2u+2v-2}\Theta^{-u+v}\{-v+u-[2uv+u(u+1)+v(v-1)]\}(\rho_1+\rho_m)(\rho_2+\rho_m) \\ &= -[AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)](r_1^C)^{-2u+2v-2}\Theta^{-u+v}(\rho_1+\rho_m)(\rho_2+\rho_m)(u+v)^2 < 0 \end{aligned}$$

Similarly, one can easily obtain

$$H_{11}H_{22} - (H_{12})^2 = [AB^{-(2\delta+1)/(\delta+1)}/(\delta+1)]^2B[BDF - (C^2F + DE^2 + 2CEL)\delta/(\delta+1) - B^2L^2] > 0$$

Hence, the matrix of Π_S is negative definite at (r_1^C, r_2^C). Due to the uniqueness of r_1^C and r_2^C, the total system profit Π_S reaches its maximum at (r_1^C, r_2^C). Therefore, $r_i^C (i = 1,2)$ and m^C given by (42)–(44) are, respectively, the optimal local advertising expenditures and brand name investment under the centralized decision system. The proof of Theorem 9 is complete. □

A.12. Derivations of (52)-(54)

Suppose the manufacturer and the two retailers are all risk averse with the following utility functions:

$$U_M(\Delta\Pi_M) = b_m\log(\Delta\Pi_M) \quad (A46)$$

$$U_{r1}(\Delta\Pi_{r1}) = b_1\log(\Delta\Pi_{r1}) \quad (A47)$$

$$U_{r2}(\Delta\Pi_{r2}) = b_2\log(\Delta\Pi_{r2}) \quad (A48)$$

The system utility function, $U_S(\Delta\Pi_M, \Delta\Pi_{r1}, \Delta\Pi_{r2})$, can be written as

$$\begin{aligned} U_S(\Delta\Pi_M, \Delta\Pi_{r1}, \Delta\Pi_{r2}) &= U_M(\Delta\Pi_M) + U_{r1}(\Delta\Pi_{r1}) + U_{r2}(\Delta\Pi_{r2}) \\ &= b_m\log(\Delta\Pi_M) + b_1\log(\Delta\Pi_{r1}) + b_2\times\log(\Delta\Pi_{r2}) \end{aligned} \quad (A49)$$

Since $\Delta\Pi_S = \Delta\Pi_M + \Delta\Pi_{r1} + \Delta\Pi_{r2}$, (A49) can be rewritten as the form in terms of $\Delta\Pi_{r1}$ and $\Delta\Pi_{r2}$:

$$U_S(\Delta\Pi_{r1}, \Delta\Pi_{r2}) = b_m\log(\Delta\Pi_S - \Delta\Pi_{r1} - \Delta\Pi_{r2}) + b_1\times\log(\Delta\Pi_{r1}) + b_2\log(\Delta\Pi_{r2}) \quad (A50)$$

By solving the first-order conditions of $U_S(\Delta\Pi_{r1}, \Delta\Pi_{r2})$ with respect to $\Delta\Pi_{r1}$ and $\Delta\Pi_{r2}$, we have

$$-b_m/(\Delta\Pi_S - \Delta\Pi_{r1} - \Delta\Pi_{r2}) + b_1/\Delta\Pi_{r1} = 0 \quad (A51)$$

$$-b_m/(\Delta\Pi_S - \Delta\Pi_{r1} - \Delta\Pi_{r2}) + b_2/\Delta\Pi_{r2} = 0 \quad (A52)$$

Using simple algebraic operations on (A51) and (A52) will yield (44)–(46).

References

Arrow, K. J. (1965). *Aspects of the theory of risk bearing*. New York: Academic Book Store.

Berger, P. D. (1972). Vertical cooperative advertising ventures. *Journal of Marketing Research, 9*, 309–312.

Berger, P. D. (1973). Statistical analysis of cooperative advertising models. *Operational Research Quarterly, 24*, 207–216.

Berger, P. D., & Magliozzi, T. (1992). Optimal co-op advertising decisions in direct-mail operations. *Journal of the Operational Research Society, 43*, 1079–1086.

Berger, P. D., Lee, J., & Weinberg, B. D. (2006). Optimal cooperative advertising integration strategy for organizations adding a direct online channel. *Journal of the Operational Research Society, 57*, 920–927.

Choi, S. C. (1991). Price competition in a channel structure with a common retailer. *Marketing Science, 10*, 271–296.

Crimmins, E. C. (1985). *Cooperative advertising*. New York: Gene Wolf.

Dant, R. P., & Berger, P. D. (1996). Modelling cooperative advertising decisions in franchising. *Journal of the Operational Research Society, 47*, 1120–1136.

Dutta, S., Bergen, M., John, G., & Rao, A. (1995). Variations in the contractual terms of cooperative advertising contracts: An empirical investigation. *Marketing Letters, 6*, 15–22.

Fulop, C. (1988). The role of advertising in the retail marketing mix. *International Journal of Advertising, 7*, 99–117.

Green, J. (2000). Still pulling the strings, but locally too. *Brandweek, 41*, 34–42.

Hariga, M. (1996). Optimal EOQ models for deteriorating items with time-varying demand. *Journal of Operational Research Society, 47*, 1228–1246.

Hempelmann, B. (2006). Optimal franchise contracts with private cost information. *International Journal of Industrial Organization, 24*, 449–465.

Huang, Z., & Li, S. X. (2001). Co-op advertising models in manufacturer–retailer supply chains: A game theory approach. *European Journal of Operational Research, 135*, 527–544.

Huang, Z., Li, S. X., & Mahajan, V. (2002). An analysis of manufacturer–retailer supply chain coordination in cooperative advertising. *Decision Sciences, 33*, 469–494.

Khouja, M., & Robbins, S. S. (2003). Linking advertising and quantity decisions in the single-period inventory model. *International Journal of Production Economics, 86*, 93–105.

Kraft, M. E., & Kamieniecki, S. (2007). *Business and environmental policy: Corporate interests in the American political system*. Cambridge: The MIT Press.

Li, S. X., Huang, Z., Zhu, J., & Chau, P. Y. K. (2002). Cooperative advertising, game theory and manufacturer–retailer supply chains. *Omega, 30*, 347–357.

Luo, P. L. (2006). *The theory of industrial organization*. Shanghai: Fudan University Press.

Nadiminti, R., Mukhopadhyay, T., & Kriebel, C. H. (1996). Risk aversion and the value of information. *Decision Support Systems, 16*(3), 241–254.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica, 32*, 122–136.

Roslow, S., Laskey, H. A., & Nicholls, J. A. F. (1993). The enigma of cooperative advertising. *Journal of Business & Industrial Marketing, 8*, 70–79.

Somers, T. M., Gupta, Y. P., & Herriott, S. R. (1990). Analysis of cooperative advertising expenditures: A transfer-function modeling approach. *Journal of Advertising Research*, 35–45.

Szmerekovsky, J. G., & Zhang, J. (2009). Pricing and two-tier advertising with one manufacturer and one retailer. *European Journal of Operational Research, 192*(3), 904–917.

Teng, J. T., Chern, M. S., & Yang, H. L. (1997). An optimal recursive method for various inventory replenishment models with increasing demand and shortages. *Naval Research Logistics, 44*, 791–806.

Wang, S. D., Zhou, Y. W., & Wang, J. P. (2010). Supply chain coordination with two production modes and random demand depending on advertising expenditure and selling price. *International Journal of Systems Science, 41*(10), 1257–1272.

Xie, J., & Ai, S. (2006). A note on "Cooperative advertising, game theory and manufacturer–retailer supply chains". *Omega, 34*, 501–504.

Xie, J., & Neyret, A. (2009). Co-op advertising and pricing models in manufacturer–retailer supply chains. *Computers and Industrial Engineering, 56*(4), 1375–1385.

Xie, J., & Wei, J. (2009). Coordinating advertising and pricing in a manufacturer–retailer channel. *European Journal of Operational Research, 197*(2), 785–791.

Young, R. F., & Greyser, S. A. (1983). *Managing cooperative advertising: A strategic approach*. Lexington, MA: Lexington Books.

Yue, J., Austin, J., Wang, M. C., & Huang, Z. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *European Journal of Operational Research, 168*, 65–85.

Zhou, Y. W., Lau, H. S., & Yang, S. L. (2003). A new variable production scheduling strategy for deteriorating items with time-varying demand and partial lost sale. *Computers & Operations Research, 30*, 1753–1776.

Zhou, Y. W., & Wang, S. D. (2009). Manufacturer–buyer coordination for newsvendor-type-products with two ordering opportunities and partial backorders. *European Journal of Operational Research, 198*(3), 958–974.