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# A note on cooperative versus non-cooperative strategies in international pollution control

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## Abstract

In this note, we evaluate the scope of Dockner and Long's [Journal of Environment Economics and Management 24 (1993) 13] conclusion on the efficiency of the non-cooperative outcome in a differential game of international pollution control. We also complete the study of the different equilibria the differential game can present. Our results show that their conclusion requires that the initial value of the stock of pollution be higher than the Pareto-efficient pollution stock so that the equilibrium path of emissions involves a decreasing stock of pollution. Our results also show that the application of the procedure proposed by Tsutsui and Mino [Journal of Economic Theory 52 (1990) 136] to construct a Markov-perfect equilibrium using non-linear strategies is problematic when the initial pollution stock is lower than the Pareto-efficient pollution stock. © 2002 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

It is usual in linear-quadratic differential games to obtain that the outcome of the linear Markov-perfect equilibrium is typically less efficient than the open-loop equilibrium outcome and far less efficient than the cooperative outcome. Tsutsui and Mino (1990) thought that this occurs because the use of linear strategies to compute the non-cooperative equilibrium de facto excludes other possible strategies which could support a more efficient outcome. For this reason they examined, for a differential game of duopolistic competition with sticky prices, whether it is possible to construct a more efficient Markov-perfect equilibrium using non-linear strategies. They found that there exist Markov-perfect equilibria

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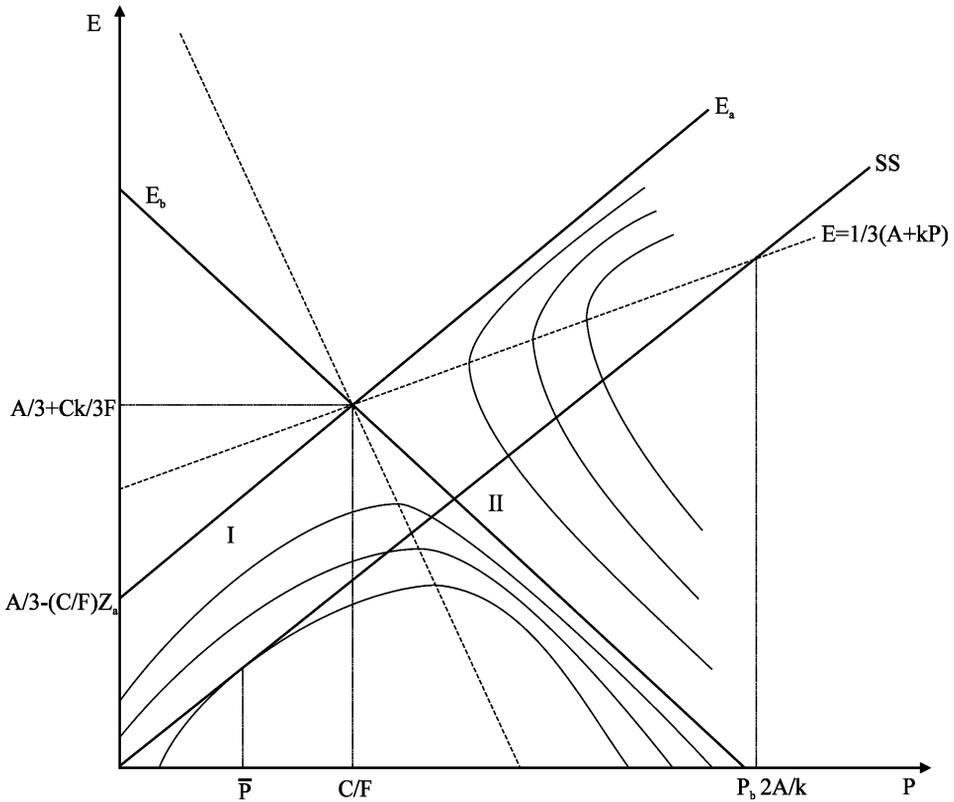


Fig. 1. Range of stable steady-states, Dockner and Long (1993).

supported by non-linear strategies which approach the cooperative outcome more than the open-loop equilibrium. Dockner and Long (1993) have obtained identical results for a differential game of international pollution control with two countries and non-linear strategies and Wirl (1994) and Wirl and Dockner (1995) have proved that cooperation between an energy cartel and a coalition of importing country governments is not necessary to reach the Pareto-efficient long-run concentration of CO<sub>2</sub> in the atmosphere.<sup>1</sup>

In this note we evaluate the generality of the result obtained by Dockner and Long “that demonstrates that the fully coordinated outcome may be approximated through an appropriate choice of non-linear Markov-perfect strategies, . . .” (p. 24).<sup>2</sup> We show that the possibility that the Pareto-efficient pollution stock level can be supported as a non-cooperative long-run equilibrium is rather one of the possible outcomes. This possibility requires not

<sup>1</sup> However, in these two papers the authors find that non-linear strategies are Pareto-inferior to the linear strategies, see Proposition 4 in Wirl (1994) and thus weaken the argument for non-linear strategies.

<sup>2</sup> In a model of pollution control with stock externalities, when players cooperate maximizing their joint welfare they internalize these externalities so that the cooperative outcome is Pareto-efficient. In Dockner and Long’s quotation, the fully coordinated outcome refers to the cooperative outcome so that the pollution stock is Pareto-efficient.

only that the rate of discount should be sufficiently small but also that the initial value of the pollution stock should belong to a restricted set of values. In particular, the initial value of the stock of pollution must be higher than the Pareto-efficient pollution stock.<sup>3</sup> We also conclude that the procedure proposed by Tsutsui and Mino to construct Markov-perfect equilibria using non-linear strategies is not useful for the differential game of international pollution control defined by Dockner and Long when the initial stock of pollution is lower than the Pareto-efficient pollution stock.

In Section 2, we summarily present the model and their main conclusion, complete the analysis of game equilibria showing the existing relationship among the different equilibria the differential game can present and make some clarifications about Fig. 1 in their paper. In Section 3 the implications of the local nature of non-linear strategies for the solutions of the game are studied and in Section 4 the linear and non-linear strategies are compared when the cost of pollution is sufficiently small. The note ends with some concluding remarks.

## 2. The model and results

There are two countries that produce a single good and emit pollutants as a by product of the production process. Emissions of pollutants,  $E(t)$ , are accumulated in the atmosphere and the natural purification is assumed to be proportional to the existing stock of pollution. That is, the stock of pollution,  $P(t)$ , evolves according to the dynamic equation:

$$\dot{P}(t) = E_1(t) + E_2(t) - kP(t), \quad k > 0 \quad (1)$$

To characterize the cooperative and non-cooperative solutions, Dockner and Long assume that the cost functions are quadratic,

$$C_i(P(t)) = \frac{1}{2}sP^2(t), \quad s > 0 \quad (2)$$

and that the utility functions are also quadratic:

$$U_i(E_i(t)) = AE_i(t) - \frac{1}{2}E_i^2(t), \quad A > 0 \quad (3)$$

so that their pollution control model becomes a linear quadratic game.

The authors demonstrate that for this game there exists a unique and asymptotically stable cooperative outcome (see Proposition 1) that results in a steady-state pollution stock given by

$$P_C = \frac{2A(r+k)}{k(r+k) + 4s} \quad (4)$$

To characterize the non-cooperative pollution control they compute, following Tsutsui and Mino (1990), a Markov-perfect equilibrium and obtain a set of non-linear strategies

<sup>3</sup> Itaya and Shimomura (2001) have addressed a similar issue in a dynamic conjectural variations model in the private provision of public goods. Their findings establish that there may be steady-state positive conjectural variations when non-linear Markov perfect strategies are used. However, in order to obtain this result an upper bound must be exogenously imposed to restrict the domain of the state variable. If this restriction is not taken into account, only the stable linear strategy is subgame perfect over the entire domain and then the steady-state conjectural variation is negative.

implicitly defined by the following equation:<sup>4</sup>

$$K = \begin{bmatrix} E - \frac{A}{3} + Z_a \frac{C}{F} - \left( Z_a + \frac{k}{3} \right) P \end{bmatrix}^{\xi_1} \begin{bmatrix} E - \frac{A}{3} + Z_b \frac{C}{F} - \left( Z_b + \frac{k}{3} \right) P \end{bmatrix}^{\xi_2} \quad (5)$$

where  $K$  is an arbitrary constant and

$$F = \frac{1}{3}(rk + k^2 + 3s), \quad C = \frac{1}{3}(2A(r + k))$$

$$Z_a = \frac{r}{6} + \sqrt{\frac{r^2}{36} + \frac{F}{3}}, \quad Z_b = \frac{r}{6} - \sqrt{\frac{r^2}{36} + \frac{F}{3}}$$

$$\xi_1 = -1 - \xi_2, \quad \xi_2 = \frac{-Z_b}{Z_b - Z_a}$$

Eq. (5) includes two singular solutions which define two linear strategies,

$$E_a = \frac{A}{3} - Z_a \frac{C}{F} + \left( Z_a + \frac{k}{3} \right) P \quad (6)$$

$$E_b = \frac{A}{3} - Z_b \frac{C}{F} + \left( Z_b + \frac{k}{3} \right) P \quad (7)$$

Dockner and Long show that linear strategies  $E_b$  lead to an asymptotically stable Markov-perfect equilibrium that results in a stock of pollution,  $P_{FL}$ , which exceeds that of the cooperative solution (see Proposition 2). Additionally, they show that any stock of pollution in the open interval

$$(P_L, P_{MY}) = \left( \frac{2A(2r + k)}{2kr + k^2 + 4s}, \frac{2A}{k} \right) \quad (8)$$

can be supported as an asymptotically stable steady-state (see Proposition 3).<sup>5</sup>

<sup>4</sup> This equation implicitly defines an uncountable number of stationary, non-linear Markov strategies, each one associated to a different value for the constant  $K$ . Thus, a Markov strategy is a decision rule that gives the level of emissions as a function of the current value of the pollution stock. In this model, the Markov strategy is stationary because it does not explicitly depend on time. Then, a stationary, non-linear Markov-perfect equilibrium is a subgame perfect Nash equilibrium for the differential game supported by stationary, non-linear Markov strategies. The subgame perfectness of the equilibrium appears when players commit themselves to a decision rule, as the one defined by (5), that is a Nash equilibrium for any value in the domain of the stock pollution. See Tsutsui and Mino (1990) and Dockner et al. (2000, Chapter 4) for more technical definitions. A referee has pointed out us that the non-uniqueness characteristic of Markov strategies for differential games result is due to a missing boundary condition of the Hamilton–Jacobi–Bellman equation. See also Tsutsui and Mino (1990, p. 153) and Dockner and Long (1993, p. 23).

<sup>5</sup> The upper bound  $2A/k$  is defined by Dockner and Long as the steady-state stock of pollution resulting for  $s$  equal to zero. This stock of pollution can also be considered as the solution to the optimization problem solved by a myopic agent who maximizes her utility without accounting for the dynamic restriction. Then, the first order condition establishes that  $E = A$ , and by substitution in (1) we get that the steady-state stock of pollution supported by the myopic strategy is  $P_{MY} = 2A/k$ . Rubio and Casino (2002) also find that the range of stable steady-states is bounded by the myopic solution for a groundwater pumping differential game.

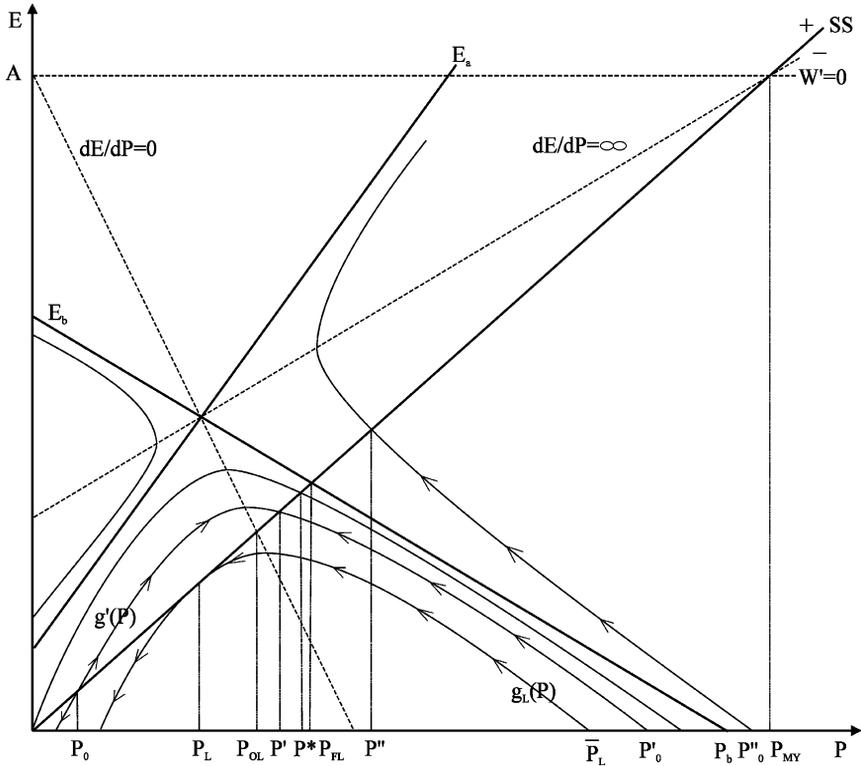


Fig. 2. Range of stable steady-states (case 1).

Then, taking into account that  $\lim_{r \rightarrow 0} P_C = P_L$ , the authors conclude that; “. . . if agents use non-linear Markov strategies and have a low discount rate, then the Pareto-efficient pollution stock levels can be supported as a long-run equilibrium. Theoretically, this is a remarkable result that demonstrates that the fully coordinated outcome may be approximated through an appropriate choice of non-linear Markov-perfect strategies, . . .” (p. 24). In this note the scope of this conclusion is assessed.<sup>6</sup>

Dockner and Long omit to represent in their figure the functional relationship,  $E = A$ , defined by the condition  $W'_i(P) = 0$  which establishes for which values of emissions the marginal utility is positive (see Fig. 2).<sup>7</sup>

<sup>6</sup> In order to facilitate to the reader the comparison between our Figure and the Figure which appears in Dockner and Long’s paper, we have reproduced this in this note (see Fig. 1). In both cases the Figure is drawn assuming that  $s$  is sufficiently large. This assumption guarantees that the unstable linear strategy,  $E_a$ , intersects with the vertical axis for a positive value.

<sup>7</sup> In Fig. 2, we have also represented the steady-state pollution stock of the open-loop Nash equilibrium,  $P_{OL}$ . In the Appendix A is shown that this steady-state is defined by the intersection point between locus  $dE/dP = 0$  and SS line. This allows us to conclude that locus  $dE/dt = 0$  associated to the open-loop Nash equilibrium coincides with locus  $dE/dP = 0$  associated to the Markov-perfect equilibrium. This coincidence seems to be a systematic relationship because it can be found in different applications of the differential games as the ones studied by Itaya and Shimomura (2001) and Rubio and Casino (2002).

Notice  $E = A$  is the strategy corresponding to the myopic equilibrium, so that the intersection point of this strategy with steady-state (SS) line defines the steady-state pollution stock corresponding to this equilibrium. Then as long as the locus  $dE/dP = 0$  intersects the vertical axis at  $A$ , the stable linear strategy of the Markov-perfect equilibrium by construction must intersect the vertical axis at a point below  $A$ . This does not happen in Fig. 1 in Dockner and Long's paper. Compare their figure with Fig. 2 presented in this note.

### 3. The implications of the local nature of non-linear strategies

In this Section, we try to clarify the scope of an important economic interpretation Dockner and Long derive from Proposition 3. "It turns out that, in the limiting case when  $r$  tends to zero, the collusive long-run pollution stock can be supported as a steady-state of non-linear differentiable Markov strategies . . . These non-linear strategies can be reached from the following domains of initial pollution stock. non-linear strategies from region I in Fig. 1 can be reached from initial conditions lying in the interval  $(0, P_b)$ . Strategies from region II can be reached by initial conditions lying in the interval  $(P_b, P_0)$  (see Fig. 1). This characterizes the local nature of the non-linear Markov strategies" (p. 23). From our point of view, this paragraph is something confusing since it suggests that the cooperative outcome can be reached from any initial value of pollution stock when in fact the cooperative steady-state can only be reached from some specific values of the initial pollution stock. The local nature of the non-linear strategies restricts the initial values of the pollution stock from which the non-cooperative solution of the game yields or approximates the collusive outcome.

In order to analyze in detail the implications of the local nature of non-linear strategies we define  $B(P_L) = [P_L, \bar{P}_L]$  as the set of initial pollution levels from which steady-state pollution stock  $P_L$  is reachable, where  $\bar{P}_L$  is given by the intersection point of integral curve  $g_L(P)$ , that is tangent to SS for the stock of pollution  $P_L$  and the horizontal axis.<sup>8</sup> Thus, if  $P_0 \in [P_L, \bar{P}_L]$  non-linear strategy  $g_L(P)$  leads to steady-state stock of pollution  $P_L$  and the cooperative outcome may be reached through an appropriate choice of non-linear Markov-perfect strategies. See our Fig. 2. Therefore, Dockner and Long's main conclusion requires that: (i)  $r$  should be sufficiently small;<sup>9</sup> and (ii)  $P_0 \in [P_L, \bar{P}_L]$ , which implies that the initial value must be higher than the efficient steady-state pollution stock. This last condition is not explicitly recognized in their

<sup>8</sup> To say that steady-state pollution stock  $P_L$  is reachable from any stock of pollution in interval  $[P_L, \bar{P}_L]$  means that for each level of pollution stock between  $P_L$  and  $\bar{P}_L$ , there exists a level of emissions given by the non-linear strategy represented by integral curve  $g_L(P)$  such that the pollution stock converges to the steady-state characterized by pollution stock  $P_L$ . This means that integral curve  $g_L(P)$ , in Fig. 2, represents the temporal path for the control and state variables that converges to steady-state pollution stock  $P_L$  from any initial pollution level in interval  $[P_L, \bar{P}_L]$ .

<sup>9</sup> Then, as we have just pointed out,  $P_C = P_L$  but in this case the steady-state is unstable. Notice that the set of asymptotically stable steady-states is defined by an open interval (see (8)), where the lower limit is exactly  $P_L$ .

paper.<sup>10</sup> A corollary of this conclusion is that the equilibrium temporal path of emissions involves a decreasing stock of pollution.

Next, we investigate what occurs if  $P_0 \notin [P_L, \bar{P}_L]$ . We find three different situations but, for all of them, we show that the steady-state stock of pollution,  $P_L$ , can be approached but it cannot be reached. For  $P_0 < P_L$  there are two possible alternatives,  $P_0$ , that is unstable and  $P'$ . This value is defined by the intersection point, on the right of  $P_L$ , of integral curve  $g'(P)$  with SS line. See again our Fig. 2. However,  $P'$  is in fact, a 'lower bound' of the stable steady-states that can be reached from  $P_0$ , because it can be approached but it can never be reached. In other words, if for  $P_0$  in Fig. 2 non-linear strategy  $g'(P)$  is selected, emissions  $E_0 = g'(P_0)$  define a point at the SS line and the stock of pollution does not change. For this reason it is necessary to select another strategy above point  $(P_0, g'(P_0))$  so that  $g(P_0) > g'(P_0)$  in order to move to the range of stable steady-states. But then the problem is, which one? Since for continuity of variables between a given  $g(P_0)$  and  $g'(P_0)$ , there exists always a strategy that gives a better approximation to the Pareto-efficient pollution stock than the strategy  $g(P)$ . Therefore,  $P'$  is finally a lower bound of the stable steady-states that can be reached from initial value  $P_0$ . Moreover, it is clear from Fig. 2 that the lower  $P_0$ , the higher the distance between  $P'$  and  $P_L$ .

If the initial value of the state variable is between  $\bar{P}_L$  and  $P_b$ , where  $P_b$  is given by the intersection point of the stable linear strategy with the horizontal axis, the nearest stable steady-state value to  $P_L$  reachable from the initial value is given by the intersection point with SS line of the integral curve which cuts the horizontal axis at the initial value. For instance, for  $P'_0$  in Fig. 2,  $P'$  is the nearest stable steady-state value to  $P_L$  that can be reached from  $P'_0$ . Thus, steady-state stock of pollution  $P'$  can be approached either from  $P_0 < P_L$  or from  $P'_0 \in (P_L, P_b)$ . However, for  $P'_0$  the non-linear strategy that leads to  $P'$  is well defined in the sense that  $P'$  can be reached from the initial value,  $P'_0$ , using non-linear strategy  $g'(P)$ . This is not the case for  $P_0$  as we have just explained. Moreover, the higher  $P_0$  in interval  $(\bar{P}_L, P_b)$ , the higher the distance between  $P'$  and  $P_L$ . So we can conclude, that if the initial value for the state variable is in interval  $(P_L, P_b)$  it is not possible to find a non-linear strategy that allows to reach steady-state value  $P_L$ . Obviously, for  $P_0 = P_b$  the nearest stable steady-state value to  $P_L$  that can be reached is the steady-state for the Markov-perfect equilibrium in linear strategies,  $P_{FL}$ .

Finally, if the state variable initial value is larger than  $P_b$ , as happens with  $P''_0$ , the nearest steady-state value to  $P_L$  is larger than  $P_{FL}$  and we can conclude as well that it is not possible to reach the cooperative outcome through a non-linear Markov-perfect strategy.

<sup>10</sup> Moreover, given the local nature of non-linear strategies the domain of the pollution stock must be restricted to interval  $[P_L, \bar{P}_L]$  to guarantee the subgame perfectness of the equilibria. See Theorem 1 in Tsutsui and Mino (1990, p. 151). In the rest of this Section it is assumed that the domain of the pollution stock is suitably redefined to guarantee the subgame perfectness. In Itaya and Shimomura (2001) the effects of this kind of restriction on the stable steady-state equilibria supported by the non-linear Markov strategies is addressed. Our paper completes the analysis defining the condition on the initial pollution stock level that guarantees that the efficient steady-state pollution stock can be reached using non-linear strategies.

**4. Linear versus non-linear strategies**

In this section we focus on the study of the long-run equilibrium of the non-cooperative game when  $s$  is not sufficiently large. If  $s$  is low enough, in particular, when  $s$  is lower than  $k^2 + 3kr + 2r^2$  the unstable linear strategy,  $E_a$ , intersects with the vertical axis for a negative value. See Fig. 3. Then, the intersection of  $E_a$  with the SS line defines unstable equilibrium  $P_a$ .

In this case, if  $P_0 < P_a$ , it is not possible to reach any value in the range of stable steady-states using non-linear strategies. In other words, non-linear Markov-perfect equilibria that support stable steady-state pollution stocks do not exist. Then, if we assume that  $P_0 < P_a$ , there exists only a strategy that leads to a stable steady-state value; the stable linear strategy  $E_b$ . Therefore, we can conclude that for  $P_0 < P_a$ , the non-cooperative long-run equilibrium of the game is supported by a linear Markov-perfect equilibrium and the Pareto-efficient pollution stock level cannot be approximated through an appropriate choice of non-linear Markov-perfect strategies. This result, completed by the one obtained in the Section 3, limits the scope of the procedure proposed by Tsutsui and

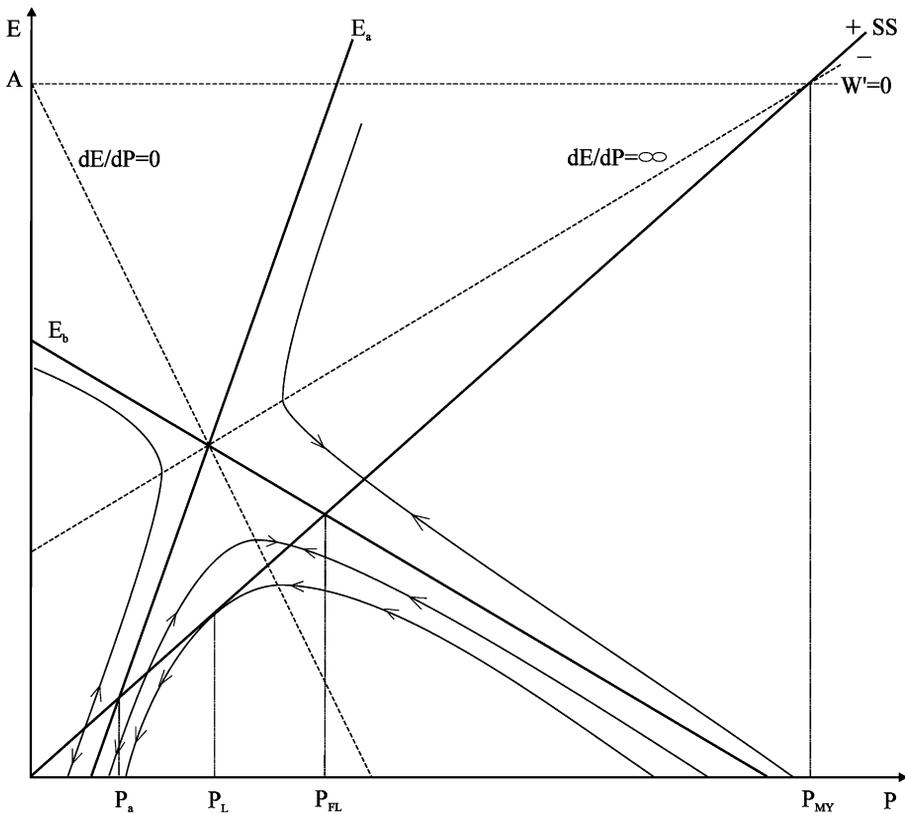


Fig. 3. Range of stable steady-states (case 2).

Mino (1990) to construct a Markov-perfect equilibrium using non-linear strategies. In this case the Markov-perfect equilibrium of the game would have to be computed using linear strategies.

The difference between the steady-state pollution stock supported by the stable linear strategy regarding the cooperative solution is given by  $\Delta = P_{FL} - P_C$  or  $\Delta = P_{FL} - P_L$  for  $r = 0$ , which can be explicitly calculated. However, we can obtain two simple expressions that set a lower and an upper bound for this difference. The lower bound is given by  $\Delta_1 = P_L - P_C$  and the upper bound is defined by the myopic solution  $\Delta_2 = P_{MY} - P_C$ , where:

$$\Delta_1 = \frac{8Ars}{(2kr + k^2 + 4s)(kr + k^2 + 4s)}$$

$$\Delta_2 = \frac{8As}{k(kr + k^2 + 4s)}$$

Both differences increase with  $A$  and decrease with  $k$  and  $\Delta_2$  also decreases as  $r$  increases and increases as  $s$  increases.

## 5. Conclusions

In this note we have tried to clarify the scope of Dockner and Long's conclusion on the efficiency of the non-cooperative outcome in a differential game of international pollution control. Our analysis shows that the possibility that the Pareto-efficient pollution stock level can be supported as a non-cooperative long-run equilibrium depends critically on the value of the initial pollution stock. The condition required to obtain this result is not only that the rate of discount has to approach zero but also that the initial value of the stock of pollution has to belong to a restricted set of values. In particular, it must be higher than the Pareto-efficient pollution stock which implies that the pollution stock must decrease along the equilibrium trajectory until reaching the steady-state. Moreover, there is nothing in the model that guarantees that the value of the initial pollution stock belongs to this restricted set of values.

On the other hand, if we assume an initial pollution stock lower than the Pareto-efficient pollution stock appears a problem to select the non-linear strategy since there exists always a strategy that gives a better approximation to the Pareto-efficient pollution stock. Moreover, with  $s$  small enough and a low initial value for the pollution stock there exist more problems since, in this case, it is not possible to reach any value in the range of stable steady-states using non-linear strategies. Only the linear strategies allow to reach a stable steady-state. These results limit the scope of the procedure proposed by Tsutsui and Mino (1990) to construct a Markov-perfect equilibrium using non-linear strategies. In fact, our conclusion is that their procedure is not useful when the initial value of the stock pollution is lower than the Pareto-efficient pollution stock.<sup>11</sup>

<sup>11</sup> We have found the same kind of problem for a groundwater pumping differential game. See Rubio and Casino (2002).

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## Appendix A. The open-loop Nash equilibrium

First, we derive the functional relationship defined by condition  $dE/dP = 0$ . By differentiating Eq. (5) we get

$$\frac{dE}{dP} = \frac{\xi_1(E - E_b)(Z_a + k/3) + \xi_2(E - E_a)(Z_b + k/3)}{\xi_1(E - E_b) + \xi_2(E - E_a)}$$

Then for  $dE/dP = 0$ , substituting  $\xi_1 = -1 - \xi_2$ , we have

$$\begin{aligned} E \left[ \xi_2(Z_b - Z_a) - Z_a - \frac{k}{3} \right] \\ = -E_b \left( Z_a + \frac{k}{3} \right) + \xi_2 \left[ E_a Z_b - E_b Z_a + (E_a - E_b) \frac{k}{3} \right] \end{aligned}$$

Now, as  $\xi_2 = -Z_b/(Z_b - Z_a)$  the above equation becomes by substitution of  $E_a$  and  $E_b$ :

$$\begin{aligned} E \left( Z_a + Z_b + \frac{k}{3} \right) = \frac{1}{Z_b - Z_a} \left\{ Z_a Z_b \left[ (Z_b - Z_a) \left( P - \frac{C}{F} \right) \right] \right. \\ \left. + \left( \frac{k}{3} P + \frac{A}{3} \right) \left[ (Z_b^2 - Z_a^2) + (Z_a - Z_b) \frac{k}{3} \right] \right\}. \end{aligned}$$

Then taking into account that  $(Z_b^2 - Z_a^2) = (Z_a + Z_b)(Z_a - Z_b)$ , we have

$$E = \frac{Z_a Z_b}{Z_a + Z_b + k/3} \left( P - \frac{C}{F} \right) + \frac{k}{3} P + \frac{A}{3}$$

Finally, by substituting  $Z_a$  and  $Z_b$  we get

$$E = A - \frac{s}{k+r} P$$

Next, we compute the open-loop Nash equilibrium. The Hamiltonian for the open-loop solution is

$$H = AE_i - \frac{1}{2} E_i^2 - \frac{1}{2} s P^2 + \lambda_i (E_i + E_j - kP), \quad i = 1, 2$$

Being the necessary conditions:

$$A - E_i + \lambda_i = 0 \tag{A.1}$$

$$\dot{\lambda}_i = (r + k)\lambda_i + sP \tag{A.2}$$

Because of symmetry we have that  $E_1 = E_2 = E$ ,  $\lambda_1 = \lambda_2 = \lambda$  and then the solution for the optimal control problem can be written in terms of a pair of differential equations:

$$\dot{E} = (r + k)(E - A) + sP \quad (\text{A.3})$$

$$\dot{P} = 2E - kP \quad (\text{A.4})$$

The locus  $\dot{E} = 0$  defines the function  $E = A - (sP)/(k + r)$  that is identical to the function defined by the locus  $dE/dP = 0$ , whereas the locus  $\dot{P} = 0$  defines the function  $E = kP/2$  that, in this case, is the steady-state line of Fig. 1 of Dockner and Long's paper. This means that the phase diagram for the system (A.3) and (A.4) is implicitly represented in Dockner and Long's Fig. 1, so that the intersection point of these two functions is the steady-state of the open-loop Nash equilibrium. Being the steady-state values

$$P_{\text{OL}}^{\infty} = \frac{2A(r + k)}{kr + k^2 + 2s}, \quad E_{\text{OL}}^{\infty} = \frac{Ak(r + k)}{kr + k^2 + 2s}$$

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